CONTINUITY AND DIFFERENTIABILITY

Single Correct Answer Type

1. Let $[x]$ denotes the greatest integer less than or equal to x and $f(x) = [\tan^2 x]$. Then,
   a) $\lim_{x \to 0} f(x)$ does not exist
   b) $f(x)$ is continuous at $x = 0$
   c) $f(x)$ is not differentiable at $x = 0$
   d) $f'(0) = 1$

2. The value of $f(0)$ so that $\frac{(-e^x + 2^x)}{x}$ may be continuous at $x = 0$ is
   a) $\log\left(\frac{1}{2}\right)$
   b) 0
   c) 4
   d) $-1 + \log 2$

3. Let $f(x)$ be an even function. Then $f'(x)$
   a) Is an even function
   b) Is an odd function
   c) May be even or odd
   d) None of these

4. If $f(x) = \begin{cases} [\cos \pi x], x < 1 \\ |x - 2|, \ 2 > x \geq 1 \end{cases}$, then $f(x)$ is
   a) Discontinuous and non-differentiable at $x = -1$ and $x = 1$
   b) Continuous and differentiable at $x = 0$
   c) Discontinuous at $x = 1/2$
   d) Continuous but not differentiable at $x = 2$

5. If $f(x) = \begin{cases} \frac{|x+2|}{\tan^{-1}(x+2)}, x \neq -2 \\ 2, \quad x = -2 \end{cases}$, then $f(x)$ is
   a) Continuous at $x = -2$
   b) Not continuous $x = -2$
   c) Differentiable at $x = -2$
   d) Continuous but not derivable at $x = -2$

6. If $f(x) = |\log|x||$, then
   a) $f(x)$ is continuous and differentiable for all x in its domain
   b) $f(x)$ is continuous for all x in its domain but not differentiable at $x = \pm 1$
   c) $f(x)$ is neither continuous nor differentiable at $x = \pm 1$
   d) None of the above

7. If $f'(a) = 2$ and $f(a) = 4$, then $\lim_{x \to a} \frac{xf(a) - af(x)}{x - a}$ equals
   a) $2a - 4$
   b) $4 - 2a$
   c) $2a + 4$
   d) None of these

8. If $f(x) = x(\sqrt{x} + \sqrt{x+1})$, then
   a) $f(x)$ is continuous but not differentiable at $x = 0$
   b) $f(x)$ is differentiable at $x = 0$
   c) $f(x)$ is not differentiable at $x = 0$
   d) None of the above

9. If $f(x) = \begin{cases} ax^2 + b, \ b \neq 0, x \leq 1 \\ x^2 b + ax + c, \ x > 1 \end{cases}$, then, $f(x)$ is continuous and differentiable at $x = 1$, if
   a) $c = 0, a = 2b$
   b) $a = b, c \in R$
   c) $a = b, c = 0$
   d) $a = b, c \neq 0$

10. For the function $f(x) = \begin{cases} |x - 3|, \ x \geq 1 \\ \frac{x^2}{4} - \frac{3x}{2} + \frac{13}{4}, \ x < 1 \end{cases}$ which one of the following is incorrect?
    a) Continuous at $x = 1$
    b) Derivable at $x = 1$
    c) Continuous at $x = 3$
    d) Derivable at $x = 3$

11. If $f: R \to R$ is defined by

$$f(x) = \begin{cases} \dfrac{2\sin x - \sin 2x}{2x\cos x}, & \text{if } x \neq 0, \\ a, & \text{if } x = 0 \end{cases}$$

Then the value of a so that f is continuous at 0 is

    a) 2   b) 1   c) -1   d) 0

12. $f(x) = x + |x|$ is continuous for

    a) $x \in (-\infty, \infty)$ b) $x \in (-\infty, \infty) - \{0\}$ c) Only $x > 0$ d) No value of x

13. If the function

$$f(x) = \begin{cases} \{1 + |\sin x|\}^{\frac{a}{|\sin x|}}, & -\dfrac{\pi}{6} < x < 0 \\ b, & x = 0 \\ e^{\frac{\tan 2x}{\tan 3x}}, & 0 < x < \dfrac{\pi}{6} \end{cases}$$

Is continuous at $x = 0$

    a) $a = \log_e b, b = \dfrac{2}{3}$ b) $b = \log_e a, a = \dfrac{2}{3}$ c) $a = \log_e b, b = 2$ d) None of these

14. If $f(x) = x^2 + \dfrac{x^2}{1+x^2} + \dfrac{x^2}{(1+x^2)^2} + \cdots + \dfrac{x^2}{(1+x^2)^n} + \cdots$, then at $x = 0, f(x)$

    a) Has no limit

    b) Is discontinuous

    c) Is continuous but not differentiable

    d) Is differentiable

15. Let $f(x) = \begin{cases} 1, & \forall & x < 0 \\ 1 + \sin x, & \forall & 0 \leq x \leq \pi/2 \end{cases}$, then what is the value of $f'(x)$ at $x = 0$?

    a) 1   b) -1 c) ∞ d) Does not exist

16. The function $f(x) = x - |x - x^2|$ is

    a) Continuous at $x = 1$ b) Discontinuous at $x = 1$

    c) Not defined at $x = 1$ d) None of the above

17. If $f(x + y + z) = f(x).f(y).f(z)$ for all x, y, z and $f(2) = 4, f'(0) = 3$, then $f'(2)$ equals

    a) 12   b) 9   c) 16   d) 6

18. If $f(x) = |\log_e |x||$, then $f'(x)$ equals

    a) $\dfrac{1}{|x|}, x \neq 0$

    b) $\dfrac{1}{x}$ for $|x| > 1$ and $\dfrac{-1}{x}$ for $|x| < 1$

    c) $\dfrac{-1}{x}$ for $|x| > 1$ and $\dfrac{1}{x}$ for $|x| < 1$

    d) $\dfrac{1}{x}$ for $|x| > 0$ and $-\dfrac{1}{x}$ for $x < 0$

19. If the function $f(x) = \begin{cases} \dfrac{1 - \cos x}{x^2}, & \text{for } x \neq 0 \\ k, & \text{for } x = 0 \end{cases}$ is continuous at $x = 0$, then the value of k is

    a) 1   b) 0   c) $\dfrac{1}{2}$ d) -1

20. Function $f(x) = |x - 1| + |x - 2|, x \in R$ is

    a) Differentiable everywhere in R

    b) Except $x = 1$ and $x = 2$ differentiable everywhere in R

    c) Not continuous at $x = 1$ and $x = 2$

    d) Increasing in R

21. The set of points where the function $f(x) = \sqrt{1 - e^{-x^2}}$ is differentiable is

    a) $(-\infty, \infty)$ b) $(-\infty, 0) \cup (0, \infty)$ c) $(-1, \infty)$ d) None of these

22. If $f(x) = x \sin\left(\dfrac{1}{x}\right), x \neq 0$, then the value of function at $x = 0$, so that the function is continuous at $x = 0$ is

a) 1   b) -1 c) 0   d) Indeterminate

23. The value of $f(0)$ so that the function $f(x) = \frac{2-(256-7x)^{1/8}}{(5x+32)^{1/5}-2}$ $(x \neq 0)$ is continuous everywhere, is given by

a) -1 b) 1   c) 26   d) None of these

24. The derivative of $f(x) = |x|^3$ at $x = 0$ is

a) -1 b) 0   c) Does not exist   d) None of these

25. If $f(x) = \begin{cases} \frac{(4^x-1)^3}{\sin\left(\frac{x}{a}\right)\log\left(1+\frac{x^2}{3}\right)}, x \neq 0 \\ 9(\log 4)^3, x = 0 \end{cases}$ is continuous function at $x = 0$, then the value of a is equal to

a) 3   b) 1   c) 2   d) 0

26. $f(x) = |[x] + x|$ in $-1 < x \leq 2$ is

a) Continuous at $x = 0$

b) Discontinuous at $x = 1$

c) Not differentiable at $x = 2, 0$

d) All the above

27. Let $f(x) = [x^3 - x]$, where $[x]$ the greatest integer function is. Then the number of points in the interval (1, 2), where function is discontinuous is

a) 4   b) 5   c) 6   d) 7

28. If $y = \cos^{-1} \cos(|x| - f(x))$, where

$f(x) = \begin{cases} 1, if \ x > 0 \\ -1, if \ x < 0. \\ 0, if \ x = 0 \end{cases}$ Then, $(dy/dx) \ x = \frac{5\pi}{4}$ is equal to

a) -1   b) 1

c) 0   d) Cannot be determined

29. Let $f(x + y) = f(x) + f(y)$ and $f(x) = x^2 g(x)$ for all $x, y \in R$, where $g(x)$ is continuous function. Then, $f'(x)$ is equal to

a) $g'(x)$ b) $g(0)$ c) $g(0) + g'(x)$ d) 0

30. Let a function $f(x)$ be defined by $f(x) = \begin{cases} x, & x \in Q \\ 0, & x \in R - Q \end{cases}$ Then, $f(x)$ is

a) Everywhere continuous

b) Nowhere continuous

c) Continuous only at some points

d) Discontinuous only at some points

31. The function $f(x) = \begin{cases} 1 - 2x + 3x^2 - 4x^3 + \cdots \text{ to } \infty, \ x \neq -1 \\ 1, \quad\quad\quad\quad\quad x = -1 \end{cases}$ is

a) Continuous and derivable at $x = -1$

b) Neither continuous nor derivable at $x = -1$

c) Continuous but not derivable at $x = -1$

d) None of these

32. $f(x) = \begin{cases} 2a - x \text{ in } -a < x < a \\ 3x - 2a \text{ in } a \leq x \end{cases}$. Then, which of the following is true?

a) $f(x)$ is discontinuous at $x = a$ b) $f(x)$ is not differentiable at $x = a$

c) $f(x)$ is differentiable at $x \geq a$ d) $f(x)$ is continuous at all $x < a$

33. Let $f(x + y) = f(x)f(y)$ and $f(x) = 1 + (\sin 2x)g(x)$ where $g(x)$ is continuous. Then, $f'(x)$ equals

a) $f(x)g(0)$ b) $2f(x)g(0)$ c) $2g(0)$ d) None of these

34. If $f(x) = [x \sin \pi x]$, then which of the following is incorrect?

a) $f(x)$ is continuous at $x = 0$

b) $f(x)$ is continuous in $(-1, 0)$

c) $f(x)$ is differentiable at $x = 1$

d) $f(x)$ is differentiable in $(-1, 1)$

35. If $f(x) = \begin{cases} 1, x < 0 \\ 1 + \sin x, 0 \le x \le \frac{\pi}{2} \end{cases}$ then derivative of $f(x)$ at $x = 0$

   a) Is equal to 1   b) Is equal to 0   c) Is equal to -1 d) Does not exist

36. If the derivative of the function $f(x)$ is everywhere continuous and is given by

   $f(x) = \begin{cases} bx^2 + ax + 4; & x \ge -1 \\ ax^2 + b; & x < -1 \end{cases}$, then

   a) $a = 2, b = -3$ b) $a = 3, b = 2$ c) $a = -2, b = -3$ d) $a = -3, b = -2$

37. If $f(x) = \begin{cases} \frac{x \log \cos x}{\log(1+x^2)}, & x \ne 0 \\ 0, & x = 0 \end{cases}$, then

   a) $f(x)$ is not continuous at $x = 0$
   b) $f(x)$ is not continuous and differentiable at $x = 0$
   c) $f(x)$ is not continuous at $x = 0$ but not differentiable at $x = 0$
   d) None of these

38. If the function $f(x) = \begin{cases} Ax - B, & x \le 1 \\ 3x, & 1 < x < 2 \\ B x^2 - A, & x \ge 2 \end{cases}$ be continuous at $x = 1$ and discontinuous at $x = 2$, then

   a) $A = 3 + B, B \ne 3$ b) $A = 3 + B, B = 3$ c) $A = 3 + B$ d) None of these

39. If $f(x) = \begin{cases} |x - 4|, \text{for } x \ge 1 \\ (x^3/2) - x^2 + 3x + (1/2), \text{for } x < 1 \end{cases}$, then

   a) $f(x)$ is continuous at $x = 1$ and $x = 4$
   b) $f(x)$ is differentiable at $x = 4$
   c) $f(x)$ is continuous and differentiable at $x = 1$
   d) $f(x)$ is not continuous at $x = 1$

40. The function $f(x) = a[x + 1] + b[x - 1]$, where $[x]$ is the greatest integer function, is continuous at $x = 1$, is

   a) $a + b = 0$ b) $a = b$ c) $2a - b = 0$ d) None of these

41. Let $f(x) = \begin{cases} 5^{1/x}, & x < 0 \\ \lambda[x], & x \ge 0 \end{cases}$ and $\lambda \in R$, then at $x = 0$

   a) f is discontinuous   b) f is continuous only, if $\lambda = 0$
   c) f is continuous only, whatever λ may be   d) None of the above

42. If for a continuous function f, $f(0) = f(1) = 0, f'(1) = 2$ and $y(x) = f(e^x)e^{f(x)}$, then $y'(0)$ is equal to

   a) 1   b) 2   c) 0   d) None of these

43. If $f(x) = \begin{cases} ax^2 - b, |x| < 1 \\ \frac{1}{|x|}, |x| \ge 1 \end{cases}$ is differentiable at $x = 1$, then

   a) $a = \frac{1}{2}, b = -\frac{1}{2}$ b) $a = -\frac{1}{2}, b = -\frac{3}{2}$ c) $a = b = \frac{1}{2}$ d) $a = b = -\frac{1}{2}$

44. Let $f(x) = \frac{\sin 4 \pi [x]}{1 + [x]^2}$, where $[x]$ is the greatest integer less than or equal to x, then

   a) $f(x)$ is not differentiable at some points
   b) $f'(x)$ exists but is different from zero
   c) $f'(x) = 0$ for all x
   d) $f'(x) = 0$ but f is not a constant function

45. The value of k which makes $f(x) = \begin{cases} \sin(1/k), & x \ne 0 \\ k, & x = 0 \end{cases}$ continuous at $x = 0$ is

   a) 8   b) 1   c) -1 d) None of these

46. The function $f(x) = \max[(1 - x), (1 + x), 2], x \in (-\infty, \infty)$ is

   a) Continuous at all points   b) Differentiable at all points
   c) Differentiable at all points except at $x = 1$ and $x = -1$ d) None of the above

47. Let $f(x)$ be defined for all $x > 0$ and be continuous. Let $f(x)$ satisfy $f\left(\frac{x}{y}\right) = f(x) - f(y)$ for all x, y and $f(e) = 1$. Then,

   a) $f(x)$ is bounded   b) $f\left(\frac{1}{x}\right) \to 0$ as $x \to 0$ c) $xf(x) \to 1$ as $x \to 0$ d) $f(x) = \ln x$

48. Suppose a function $f(x)$ satisfies the following conditions for all x and y: (i) $f(x + y) = f(x)f(y)$ (ii) $f(x) = 1 + x\,g(x)\log a$, where $a > 1$ and $\lim_{x \to 0} g(x) = 1$. Then, $f'(x)$ is equal to

   a) $\log a$ b) $\log a^{f(x)}$ c) $\log(f(x))^a$ d) None of these

49. Let $g(x)$ be the inverse of the function $f(x)$ and $f'(x) = \frac{1}{1+x^3}$. Then, $g'(x)$ is equal to

   a) $\dfrac{1}{1 + (g(x))^3}$ b) $\dfrac{1}{1 + (f(x))^3}$ c) $1 + (g(x))^3$ d) $1 + (f(x))^3$

50. If $f(x) = |x^2 - 4x + 3|$, then
   a) $f'(1) = -1$ and $f'(3) = 1$
   b) $f'(1) = -1$ and $f'(3)$ does not exist
   c) $f'(1) = -1$ does not exist and $f'(3) = 1$
   d) Both $f'(1)$ and $f'(3)$ do not exist

51. The points of discontinuity of $\tan x$ are

   a) $n\pi, n \in I$ b) $2n\pi, n \in I$ c) $(2n + 1)\frac{\pi}{2}, n \in I$ d) None of these

52. Let $f(x) = ||x| - 1|$, then points where $f(x)$ is not differentiable, is/(are)
   a) $0, \pm 1$ b) ± 1 c) 0 d) 1

53. $f(x) = \begin{cases} 2x, & x < 0 \\ 2x + 1, & x \geq 0 \end{cases}$. Then

   a) $f(x)$ is continuous at $x = 0$ b) $f(|x|)$ is continuous at $x = 0$ c) $f(x)$ is discontinuous at $x = 0$ d) None of the above

54. Let $f(x) = [x] + \sqrt{x - [x]}$, where $[x]$ denotes the greatest integer function. Then,
   a) $f(x)$ is continuous on R^+
   b) $f(x)$ is continuous on R
   c) $f(x)$ is continuous on $R - Z$
   d) None of these

55. The function $f(x) = \frac{1 - \sin x + \cos x}{1 + \sin x + \cos x}$ is not defined at $x = \pi$. The value of $f(\pi)$, so that $f(x)$ is continuous at $x = \pi$, is
   a) -1/2   b) ½   c) -1   d) 1

56. Let $f(x) = \begin{cases} (x - 1) \sin \frac{1}{x-1}, & \text{if } x \neq 1 \\ 0, & \text{if } x = 1 \end{cases}$. Then, which one of the following is true?
   a) f is differentiable at $x = 1$ but not at $x = 0$
   b) f is neither differentiable at $x = 0$ nor at $x = 1$
   c) f is differentiable at $x = 0$ and at $x = 1$
   d) f is differentiable at $x = 0$ but not at $x = 1$

57. If $f(x) = \begin{cases} mx + 1, & x \leq \frac{\pi}{2} \\ \sin x + n, & x > \frac{\pi}{2} \end{cases}$ is continuous at $x = \frac{\pi}{2}$, then

   a) $m = 1, n = 0$ b) $m = \frac{n\pi}{2} + 1$ c) $n = \frac{m\pi}{2}$ d) $m = n = \frac{\pi}{2}$

58. Let f be differentiable for all x. If $f(1) = -2$ and $f'(x) \geq 2$ for $x \in [1, 6]$, then
   a) $f(6) = 5$ b) $f(6) < 5$ c) $f(6) < 8$ d) $f(6) \geq 8$

59. If $\lim_{x \to a^+} f(x) = l = \lim_{x \to a^-} g(x)$ and $\lim_{x \to a^-} f(x) = m \lim_{x \to a^+} g(x)$, then the function $f(x)\,g(x)$
   a) Is not continuous at $x = a$
   b) Has a limit when $x \to a$ and it is equal to lm

c) Is continuous at $x = a$

d) Has a limit when $x \to a$ but it is not equal to lm

60. Let $f(x)$ be a function satisfying $f(x + y) = f(x)f(y)$ for all $x, y \in R$ and $f(x) = 1 + x \, g(x)$ where $\lim_{x \to 0} g(x) = 1$. Then, $f'(x)$ is equal to

a) $g'(x)$ b) $g(x)$ c) $f(x)$ d) None of these

61. The set of points where the function $f(x) = x|x|$ is differentiable is

a) $(-\infty, \infty)$ b) $(-\infty, 0) \cup (0, \infty)$ c) $(0, \infty)$ d) $[0, \infty)$

62. If $f(x + y) = f(x)f(y)$ for all real x and y, $f(6) = 3$ and $f'(0) = 10$, then $f'(6)$ is

a) 30   b) 13   c) 10   d) 0

63. If $f(x) = |x - a|\phi(x)$, where $\phi(x)$ is continuous function, then

a) $f'(a^+) = \phi(a)$ b) $f'(a^-) = \phi(a)$ c) $f'(a^+) = f'(a^-)$ d) None of these

64. If $f(x) = \begin{cases} xe^{-\left(\frac{1}{|x|} + \frac{1}{x}\right)}, & x \neq 0 \\ 0, & x = 0 \end{cases}$, then $f(x)$ is

a) Continuous as well as differentiable for all x

b) Continuous for all x but not differentiable at $x = 0$

c) Neither differentiable nor continuous at $x = 0$

d) Discontinuous everywhere

65. If $f(x) = \begin{cases} 3, & x < 0 \\ 2x + 1, & x \geq 0 \end{cases}$, then

a) Both $f(x)$ and $f(|x|)$ are differentiable at $x = 0$

b) $f(x)$ is differentiable but $f(|x|)$ is not differentiable at $x = 0$

c) $f(|x|)$ is differentiable but $f(x)$ is not differentiable at $x = 0$

d) Both $f(x)$ and $f(|x|)$ are not differentiable at $x = 0$

66. If $\lim_{x \to c} \frac{f(x) - f(c)}{x - c}$ exists finitely, then

a) $\lim_{x \to c} f(x) = f(c)$

b) $\lim_{x \to c} f'(x) = f'(c)$

c) $\lim_{x \to c} f(x)$ does not exist

d) $\lim_{x \to c} f(x)$ may or may not exist

67. The number of points at which the function $f(x) = |x - 0.5| + |x - 1| + \tan x$ does not have a derivative in the interval $(0, 2)$, is

a) 1   b) 2   c) 3   d) 4

68. If $f(x) = \begin{cases} \log_{(1-3x)}(1 + 3x), & \text{for } x \neq 0 \\ k, & \text{for } x = 0 \end{cases}$ is continuous at $x = 0$, then k is equal to

a) -2   b) 2   c) 1   d) -1

69. Let $f(x)$ be a function differentiable at $x = c$. Then, $\lim_{x \to c} f(x)$ equals

a) $f'(c)$ b) $f''(c)$ c) $\frac{1}{f(c)}$ d) None of these

70. If $f(x) = ae^{|x|} + b|x|^2$; $a, b \in R$ and $f(x)$ is differentiable at $x = 0$. Then a and b are

a) $a = 0, b \in R$ b) $a = 1, b = 2$ c) $b = 0, a \in R$ d) $a = 4, b = 5$

71. Let $f(x) = (x + |x|)|x|$. The, for all x

a) f and f' are continuous

b) f is differentiable for some x

c) f' is not continuous

d) f'' is continuous

72. If $f(x) = \begin{cases} \frac{x-1}{2x^2 - 7x + 5}, & \text{for } x \neq 1 \\ -\frac{1}{3}, & \text{for } x = 1 \end{cases}$, then $f'(1)$ is equal to

a) $-\frac{1}{9}$ b) $-\frac{2}{9}$ c) $-\frac{1}{3}$ d) $\frac{1}{3}$

73. Suppose $f(x)$ is differentiable at $x = 1$ and $\lim_{h \to 0} \frac{1}{h} f(1 + h) = 5$, then $f'(1)$ equals

a) 6   b) 5   c) 4   d) 3

74. If $f: R \to R$ is defined by

$$f(x) = \begin{cases} \frac{x+2}{x^2+3x+2}, & \text{if } x \in R - \{-1, -2\} \\ -1, & \text{if } x = -2 \\ 0, & \text{if } x = -1 \end{cases}$$, then f is continuous on the set

a) R b) $R - \{-2\}$ c) $R - \{-1\}$ d) $R - (-1, -2)$

75. Let $f(x) = \frac{(e^x - 1)^2}{\sin\left(\frac{x}{a}\right) \log\left(1 + \frac{x}{4}\right)}$ for $x \neq 0$ and $f(0) = 12$. If f is continuous at $x = 0$, then the value of a is equal to

a) 1   b) -1   c) 2   d) 3

76. If a function $f(x)$ is given by $f(x) = \frac{x}{1+x} + \frac{x}{(x+1)(2x+1)} + \frac{x}{(2x+1)(3x+1)} + \cdots \infty$ then at $x = 0$, $f(x)$

a) Has no limit

b) Is not continuous

c) Is continuous but not differentiable

d) Is differentiable

77. If $f(x)$ is continuous function and $g(x)$ be discontinuous, then

a) $f(x) + g(x)$ must be continuous

b) $f(x) + g(x)$ must be discontinuous

c) $f(x) + g(x)$ for all x

d) None of these

78. A function $f: R \to R$ satisfies the equation $f(x + y) = f(x)f(y)$ for all $x, y \in R$ and $f(x) \neq 0$ for all $x \in R$. If $f(x)$ is differentiable at x = 0 and $f'(0) = 2$, then $f'(x)$ equals

a) $f(x)$ b) $-f(x)$ c) $2f(x)$ d) None of these

79. Consider $f(x) = \begin{cases} \frac{x^2}{|x|}, & x \neq 0 \\ 0, & x = 0 \end{cases}$

a) $f(x)$ is discontinuous everywhere

b) $f(x)$ is continuous everywhere

c) $f'(x)$ exists in $(-1, 1)$

d) $f'(x)$ exists in $(-2, 2)$

80. If $f(x)$ is continuous at $x = 0$ and $f(0) = 2$, then

$\lim_{x \to 0} \frac{\int_0^x f(u) du}{x}$ is

a) 0   b) 2   c) $f(2)$ d) None of these

81. Let $f(x + y) = f(x)f(y)$ for all $x, y \in R$. Suppose that $f(3) = 3$ and $f'(0) = 11$ then, $f'(3)$ is equal to

a) 22   b) 44   c) 28   d) None of these

82. If $f(x) = \begin{cases} x - 5, & \text{for } x \leq 1 \\ 4x^2 - 9, & \text{for } 1 < x < 2 \\ 3x + 4, & \text{for } x \geq 2 \end{cases}$, then $f'(2^+)$ is equal to

a) 0   b) 2   c) 3   d) 4

83. $f(x) = \sin |x|$. Then $f(x)$ is not differentiable at

a) $x = 0$ only   b) All x c) Multiples of π d) Multiples of $\frac{\pi}{2}$

84. If $f(x) = \sum_{n=0}^{\infty} \frac{x^n}{n!} (\log_e a)^n, a > 0, a \neq 0$, then at $x = 0$, $f(x)$ is

a) Everywhere continuous but not differentiable

b) Everywhere differentiable

c) Nowhere continuous

d) None of these

85. The function $f(x) = [x] \cos \left[\frac{2x-1}{2}\right] \pi$ where [.] denotes the greatest integer function, is discontinuous at

   a) All x

   b) No x

   c) All integer points

   d) x which is not an integer

86. The function $f(x) = \begin{cases} 1, & |x| \geq 1 \\ \frac{1}{n^2}, \frac{1}{n} < |x| < \frac{1}{n-1}, n = 2, 3, \ldots \\ 0, & x = 0 \end{cases}$

   a) Is discontinuous at finitely many points

   b) Is continuous everywhere

   c) Is discontinuous only at $x = \pm \frac{1}{n}, n \in Z - \{0\}$ and $x = 0$

   d) None of these

87. Let f is a real-valued differentiable function satisfying $|f(x) - f(y)| \leq (x-y)^2$, $x, y \in R$ and $f(0) = 0$, then $f(1)$ equals

   a) 1

   b) 2

   c) 0

   d) -1

88. Let $f(x) = [2x^3 - 5]$, $[]$ denotes the greatest integer function. Then number of points $(1, 2)$ where the function is discontinuous, is

   a) 0

   b) 13

   c) 10

   d) 3

89. ln[1, 3] the function $[x^2 + 1]$, $[x]$ denoting the greatest integer function, is continuous

   a) For all x

   b) For all x except at four points

   c) For all except at seven points

   d) For all except at eight-points

90. If $f(x) = |\log_{10} x|$, then at $x = 1$

   a) $f(x)$ is continuous and $f'(1^+) = \log_{10} e$, $f'(1^-) = -\log_{10} e$

   b) $f(x)$ is continuous and $f'(1^+) = \log_{10} e$, $f'(1^-) = \log_{10} e$

   c) $f(x)$ is continuous and $f'(1^-) = \log_{10} e$, $f'(1^+) = -\log_{10} e$

   d) None of these

91. The function $f(x) = |\cos x|$ is

   a) Everywhere continuous and differentiable

   b) Everywhere continuous and but not differentiable at $(2n + 1) \pi/2, n \in Z$

   c) Neither continuous nor differentiable at $(2n + 1) \pi/2, n \in Z$

   d) None of these

92. Let $f(x) = \begin{cases} \frac{x-4}{|x-4|} + a, x < 4 \\ a + b, & x = 4 \\ \frac{x-4}{|x-4|} + b, x > 4 \end{cases}$

   Then, $f(x)$ is continuous at $x = 4$ when

   a) $a = 0, b = 0$

   b) $a = 1, b = 1$

   c) $a = -1, b = 1$

   d) $a = 1, b = -1$

93. If $f(x) = \begin{cases} \frac{2^x - 1}{\sqrt{1+x}-1}, -1 \leq x < \infty, \ x \neq 0 \\ k, & x = 0 \end{cases}$ is continuous everywhere, then k is equal to

   a) $\frac{1}{2} \log_e 2$

   b) $\log_e 4$

   c) $\log_e 8$

   d) $\log_e 2$

94. The function $f(x) = \begin{cases} x^n \sin\left(\frac{1}{x}\right), x \neq 0 \\ 0, \ x = 0 \end{cases}$ is continuous and differentiable at $x = 0$, if

   a) $n \in (0, 1]$

   b) $n \in [1, \infty)$

   c) $n \in (1, \infty)$

   d) $n \in (-\infty, 0)$

95. The function $f(x) = \begin{cases} \frac{e^{1/x}-1}{e^{1/x}+1}, x \neq 0 \\ 0, \ x = 0 \end{cases}$

   a) Is continuous at $x = 0$

   b) Is not continuous at $x = 0$

c) Is not continuous at $x = 0$, but can be made continuous $x = 0$

d) None of these

96. A function $f(x) = \begin{cases} 1 + x, & x \leq 2 \\ 5 - x, & x > 2 \end{cases}$ is

a) Not continuous at $x = 2$　　　　　　　　b) Differenti8able at $x = 2$

c) Continuous but not differentiable at $= 2$　　d) None of the above

97. Let $f(x + y) = f(x)f(y)$ for all $x, y \in R$. If $f'(1) = 2$ and $f(4) = 4$, then $f'(4)$ equal to

a) 4　　　　　　b) 1　　　　　　c) 1/2　　　　　　d) 8

98. Let $f(x) = [x]$ and $g(x) = \begin{cases} 0, & x \in Z \\ x^2, & x \in R - Z \end{cases}$ Then, which one of the following is incorrect?

a) $\lim\limits_{x \to 1} g(x)$ exists, but $g(x)$ is not continuous at $x = 1$

b) $\lim\limits_{x \to 1} f(x)$ does not exist and $f(x)$ is not continuous at $x = 1$

c) gof is continuous for all x

d) fog is continuous for all x

99. If $f(x) = \begin{cases} x, & \text{for} & 0 < x < 1 \\ 2 - x, & \text{for} & 1 \leq x < 2 \\ x - (1/2)x^2, & \text{for} & x = 2 \end{cases}$.Then, $f'(1)$ is equal to

a) -1　　　　　　b) 1　　　　　　c) 0　　　　　　d) None of these

100. The function $f(x) = |x| + \frac{|x|}{x}$ is

a) Discontinuous at origin because $|x|$ is discontinuous there

b) Continuous at origin

c) Discontinuous at origin because both $|x|$ and $\frac{|x|}{x}$ are discontinuous there

d) Discontinuous at the origin because $\frac{|x|}{x}$ is discontinuous there

101. $f(x) = |x - 3|$ is ... at $x = 3$

a) Continuous and not differentiable　　　b) Continuous and differentiable

c) Discontinuous and not differentiable　　d) Discontinuous and differentiable

102. At $x = \frac{3}{2}$ the function $f(x) = \frac{|2x-3|}{2x-3}$ is

a) Continuous　　b) Discontinuous　　c) Differentiable　　d) Non-zero

103. The following functions are differentiable on $(-1, 2)$

a) $\int\limits_{x}^{2x} (\log t)^2 dt$　　b) $\int\limits_{x}^{2x} \frac{\sin t}{t} dt$　　c) $\int\limits_{x}^{2x} \frac{1 - t + t^2}{1 + t + t^2} dt$　　d) None of these

104. Let $f(x) = \frac{1 - \tan x}{4x - \pi}, x \neq \frac{\pi}{4}, x \in \left[0, \frac{\pi}{2}\right]$. If $f(x)$ is continuous in $\left[0, \frac{\pi}{2}\right]$, then $f\left(\frac{\pi}{4}\right)$ is

a) 1　　　　　　b) 1/2　　　　　　c) -1/2　　　　　　d) -1

105. If $f(x) = \begin{cases} \frac{1 - \cos x}{x}, & x \neq 0 \\ k, & x = 0 \end{cases}$ is continuous at $x = 0$, then the value of k is

a) 0　　　　　　b) $\frac{1}{2}$　　　　　　c) $\frac{1}{4}$　　　　　　d) $-\frac{1}{2}$

106. Let $f(x) = |x| + |x - 1|$, then

a) $f(x)$ is continuous at $x = 0$, as well as at $x = 1$

b) $f(x)$ is continuous at $x = 0$, but not at $x = 1$

c) $f(x)$ is continuous at $x = 1$, but not at $x = 0$

d) None of these

107. The function $f(x)$ is defined as $f(x) = \frac{2x - \sin^{-1} x}{2x + \tan^{-1} x}$, if $x \neq 0$. The value of f to be assigned at $x = 0$ so that the function is continuous there, is

a) $-\frac{1}{3}$　　　　　　b) 1　　　　　　c) $\frac{2}{3}$　　　　　　d) $\frac{1}{3}$

108. Let $f(x)$ be an odd function. Then $f'(x)$
   a) Is an even function   b) Is an odd function   c) May be even or odd   d) None of these

109. If $f(x) = \begin{cases} \frac{x-1}{2x^2-7x+5}, & \text{for } x \neq 1 \\ -\frac{1}{3}, & \text{for } x = 1 \end{cases}$, then $f'(1)$ is equal to
   a) $-\frac{1}{9}$ b) $-\frac{2}{9}$ c) -13 d) $1/3$

110. If $f: R \to R$ given by

   $f(x) = \begin{cases} 2\cos x, & \text{if } x \leq -\frac{\pi}{2} \\ a + \sin x + b, & \text{if } -\frac{\pi}{2} < x < \frac{\pi}{2} \\ 1 + \cos^2 x, & \text{if } x \geq \frac{\pi}{2} \end{cases}$ is a continuous

   Function on R, then (a, b) is equal to
   a) $(1/2, 1/2)$ b) $(0, -1)$ c) $(0, 2)$ d) $(1, 0)$

111. If $f(x + y) = f(x)f(y)$ for all $x, y \in R, f(5) = 2, f'(0) = 3$. Then $f'(5)$ equals
   a) 6   b) 3   c) 5   d) None of these

112. Let $f(x)$ be a function satisfying $f(x + y) = f(x) + f(y)$ and $f(x) = x\, g(x)$ for all $x, y \in R$, where $g(x)$ is continuous. Then,
   a) $f'(x) = g'(x)$ b) $f'(x) = g(x)$ c) $f'(x) = g(0)$ d) None of these

113. If $f(x) = \sqrt{x + 2\sqrt{2x - 4}} + \sqrt{x - 2\sqrt{2x - 4}}$, then $f(x)$ is differentiable on
   a) $(-\infty, \infty)$ b) $[2, \infty) - \{4\}$ c) $[2, \infty)$ d) None of these

114. If $f(x) = \begin{cases} x^2 \sin\left(\frac{1}{x}\right), & x \neq 0 \\ 0, & x = 0 \end{cases}$, then
   a) f and f' are continuous at $x = 0$
   b) f is derivable at $x = 0$ and f' is continuous at $x = 0$
   c) f is derivable at $x = 0$ and f' is not continuous at $x = 0$
   d) f' is derivable at $x = 0$

115. If a function $f(x)$ is defined as $f(x) = \begin{cases} \frac{x}{\sqrt{x^2}}, & x \neq 0 \\ 0, & x = 0 \end{cases}$ then
   a) $f(x)$ is continuous at $x = 0$ but not differentiable at $x = 0$
   b) $f(x)$ is continuous as well as differentiable at $x = 0$
   c) $f(x)$ is discontinuous at $x = 0$
   d) None of these

116. If $f(x) = [\sqrt{2} \sin x]$, where $[x]$ represents the greatest integer function, then
   a) $f(x)$ is periodic
   b) Maximum value of $f(x)$ is 1 in the interval $[-2\pi, 2\pi]$
   c) $f(x)$ is discontinuous at $x = \frac{n\pi}{2} + \frac{\pi}{4}, n \in Z$
   d) $f(x)$ is differentiable at $x = n\pi, n \in Z$

117. $\lim_{x \to 0}[(1 + 3x)^{1/x}] = k$, then for continuity at $x = 0, k$ is
   a) 3   b) -3   c) e^3 d) e^{-3}

118. Let $f(x) = \begin{cases} \int_0^x \{5 + |1 - t|\}dt, & \text{if } x > 2 \\ 5x + 1, & \text{if } x \leq 2 \end{cases}$
   a) $f(x)$ is continuous at $x = 2$
   b) $f(x)$ is continuous but not differentiable at $x = 2$
   c) $f(x)$ is everywhere differentiable
   d) The right derivative of $f(x)$ at $x = 2$ does not exist

119. Let $f(x) = \begin{cases} \frac{1}{|x|} & \text{for } |x| \geq 1 \\ ax^2 + b & \text{for } |x| < 1 \end{cases}$

If $f(x)$ is continuous and differentiable at any point, then

a) $a = \frac{1}{2}, b = -\frac{3}{2}$
b) $a = -\frac{1}{2}, b = \frac{3}{2}$
c) $a = 1, b = -1$
d) None of these

120. If function $f(x) = \begin{cases} x, & \text{if } x \text{ is rational} \\ 1 - x, & \text{if } x \text{ is irrational} \end{cases}$, then the number of points at which $f(x)$ is continuous, is

a) ∞
b) 1
c) 0
d) None of these

121. The function $f(x) = e^{-|x|}$ is

a) Continuous everywhere but not differentiable at $x = 0$
b) Continuous and differentiable everywhere
c) Not continuous at $x = 0$
d) None of the above

122. The value of $f(0)$, so that the function

$$f(x) = \frac{\sqrt{a^2 - ax + x^2} - \sqrt{a^2 + ax + x^2}}{\sqrt{a + x} - \sqrt{a - x}}$$

Becomes continuous for all x, is given by

a) $a^{3/2}$
b) $a^{1/2}$
c) $-a^{1/2}$
d) $-a^{3/2}$

123. The value of k for which the function

$$f(x) = \begin{cases} \frac{1 - \cos 4x}{8x^2}, & x \neq 0 \\ k & x = 0 \end{cases} \text{ is continuous at } a = 0, \text{ is}$$

a) $k = 0$
b) $k = 1$
c) $k = -1$
d) None of these

124. The number of points at which the function $f(x) = (|x - 1| + |x - 2| + \cos x)$ where $x \in [0, 4]$ is not continuous, is

a) 1
b) 2
c) 3
d) 0

125. If $f(x) = \begin{cases} x \sin \frac{1}{x}, & x \neq 0 \\ k, & x = 0 \end{cases}$ is continuous at $x = 0$, then the value of k is

a) 1
b) -1
c) 0
d) 2

126. Let $f(x)$ be twice differentiable function such that $f''(x) = -f(x)$ and $f'(x) = g(x), h(x) = \{f(x)\}^2 + \{g(x)\}^2$. If $h(5) = 11$, then $h(10)$ is equal to

a) 22
b) 11
c) 0
d) None of these

127. if $f(x) = |x|^3$, then $f'(0)$ equals

a) 0
b) 1/2
c) -1
d) -1/2

128. Let function $f(x) = \sin^{-1}(\cos x)$, is

a) Discontinuous at $x = 0$
b) Continuous at $x = 0$
c) Differentiable at $x = 0$
d) None of these

129. Let $f(x) = \begin{cases} \frac{x^4 - 5x^2 + 4}{|(x-1)(x-2)|}, & x \neq 1, 2 \\ 6, & x = 10 \\ 12, & x = 2 \end{cases}$ Then, $f(x)$ is continuous on the set

a) R
b) $R - \{1\}$
c) $R - \{2\}$
d) $R - \{1, 2\}$

130. The set of points, where $f(x) = \frac{x}{1+|x|}$ is differentiable, is

a) $(-\infty, -1) \cup (-1, \infty)$
b) $(-\infty, \infty)$
c) $(0, \infty)$
d) $(-\infty, 0) \cup (0, \infty)$

131. Given $f(0) = 0$ and $f(x) = \frac{1}{(1 - e^{-1/x})}$ for $x \neq 0$. Then only one of the follo0wing statements on $f(x)$ is true.

That id $f(x)$, is

a) Continuous at $x = 0$
b) Not continuous at $x = 0$

c) Both continuous and differentiable at $x = 0$

d) Not defined at $x = 0$

132. Let f and g be differentiable functions satisfying $g'(a) = 2, g(a) = b$ and $fog = I$ (identify function). Then, $f'(b)$ is equal to

a) 1/2   b) 2   c) 2/3   d) None of these

133. Let $f(x) = \begin{cases} \frac{\sin \pi x}{5x}, & x \neq 0 \\ k, & x = 0 \end{cases}$, if $f(x)$ is continuous at $x = 0$, then k is equal to

a) $\frac{\pi}{5}$ b) $\frac{5}{\pi}$ c) 1   d) 0

134. The number of discontinuities of the greatest integer function $f(x) = [x], x \in \left(-\frac{7}{2}, 100\right)$ is equal to

a) 104   b) 100   c) 102   d) 103

135. For the function $f(x) = \frac{e^{1/x}-1}{e^{1/x}+1}, x = 0$, which of the following is correct?

a) $\lim_{x \to 0} f(x)$ does not exist

b) $\lim_{x \to 0} f(x) = 1$

c) $\lim_{x \to 0} f(x)$ exists but $f(x)$ is not continuous at $x = 0$

d) $f(x)$ is continuous at $x = 0$

136. If $f(x) = x^4 + \frac{x^4}{1+x^4} + \frac{x^4}{(1+x^4)^2} + \cdots$ to ∞ then at $x = 0, f(x)$ is

a) Continuous but not differentiable

b) Differentiable

c) Continuous

d) None of these

137. If $f(x) = \begin{cases} 1 + x, & 0 \leq x \leq 2 \\ 3 - x, & 2 < x \leq 3 \end{cases}$ then the set of points of discontinuity of $g(x) = fof(x)$, is

a) $\{1, 2\}$ b) $\{0, 1, 2\}$ c) $\{0, 1\}$ d) None of these

138. Let $g(x)$ be the inverse of an invertible function $f(x)$ which is differentiable at $x = c$, then $g'(f(c))$ equals

a) $f'(c)$ b) $\frac{1}{f'(c)}$ c) $f(c)$ d) None of these

139. If $f(x) = \begin{cases} x^p \cos\left(\frac{1}{x}\right), & x \neq 0 \\ 0, & x = 0 \end{cases}$ is differentiable at $x = 0$, then

a) $p < 0$ b) $0 < p < 1$ c) $p = 1$ d) $p > 1$

140. At $x = 0$, the function $f(x) = |x|$ is

a) Continuous but not differentiable   b) Discontinuous and differentiable

c) Discontinuous and not differentiable   d) Continuous and differentiable

141. If $f(x) = \begin{cases} (x-2)^2 \sin\left(\frac{1}{x-2}\right) - |x-1|, & x \neq 2 \\ -1, & x = 2 \end{cases}$ then the set of points where $f(x)$ is differentiable, is

a) R b) $R - \{1, 2\}$ c) $R - \{1\}$ d) $R - \{2\}$

142. The value of f at $x = 0$ so that function $f(x) = \frac{2^x - 2^{-x}}{x}, x \neq 0$ is continuous at $x = 0$, is

a) 0   b) $\log 2$ c) 4   d) $\log 4$

143. If $f(x) = |\log_e x|$, then

a) $f'(1^+) = 1, f'(1^-) = -1$

b) $f'(1^-) = -1, f'(1^+) = 0$

c) $f'(1) = 1, f'(1^-) = 0$

d) $f'(1) = -1, f'(1^+) = -1$

144. Let $f(x)$ be a function such that $f(x + y) = f(x) + f(y)$ and $f(x) = \sin x\, g(x)$ for all $x, y \in R$. If $g(x)$ is a continuous function such that $g(0) = k$, then $f'(x)$ is equal to

a) k b) kx c) $kg(x)$ d) None of these

145. The function $f(x) = |x| + |x-1|$, is
    a) Continuous at $x = 1$, but not differentiable
    b) Both continuous and differentiable at $x = 1$
    c) Not continuous at $x = 1$
    d) None of these

146. The set of points of differentiability of the function $f(x) = \begin{cases} \frac{\sqrt{x+1}-1}{x}, & \text{for } x \neq 0 \\ 0, & \text{for } x = 0 \end{cases}$ is

    a) R b) $[0, \infty]$ c) $(-\infty, 0)$ d) $R - \{0\}$

147. Given that $f(x)$ is a differentiable function of x and that $f(x) \cdot f(y) = f(x) + f(y) + f(xy) - 2$ and that $f(2) = 5$. Then, $f(3)$ is equal to
    a) 10   b) 24   c) 15   d) None of these

148. If $f(x) = \frac{1}{2} x - 1$, then on the interval $[0, \pi]$,

    a) $\tan[f(x)]$ and $\frac{1}{f(x)}$ are both continuous

    b) $\tan[f(x)]$ and $\frac{1}{f(x)}$ are both discontinuous

    c) $\tan[f(x)]$ and $f^{-1}(x)$ are both continuous

    d) $\tan[f(x)]$ s continuous but $\frac{1}{f(x)}$ is not

149. If $f(x) = (x+1)^{\cot x}$ be continuous at $= 0$, then $f(0)$ is equal to
    a) 0   b) $-e$ c) e d) None of these

150. Let $f(x) = \begin{cases} \frac{\tan x - \cot x}{x - \frac{\pi}{4}}, & x \neq \frac{\pi}{4} \\ a, & x = \frac{\pi}{4} \end{cases}$ the value of a so that $f(x)$ is continuous at $x = \frac{\pi}{4}$ is

    a) 2   b) 4   c) 3   d) 1

151. If $f(x) = \int_{-1}^{x} |t| \, dt, x \geq -1$, then

    a) f and f' are continuous for $x + 1 > 0$
    b) f is continuous but f' is not so for $x + 1 > 0$
    c) f and f' are continuous at $x = 0$
    d) f is continuous at $x = 0$ but f' is not so

152. The set of points of discontinuity of the function
    $f(x) = \lim\limits_{n \to \infty} \frac{x^{-n} - x^n}{x^{-n} + x^n}, n \in Z$ is

    a) $\{1\}$ b) $\{-1\}$ c) $\{-1, 1\}$ d) None of these

153. The number of points of discontinuity of the function
    $f(x) = \frac{1}{\log |x|}$, is

    a) 4   b) 3   c) 2   d) 1

154. $f(x) = \begin{cases} \frac{\sin 3x}{\sin x}, & x \neq 0 \\ k, & x = 0 \end{cases}$ is continuous, if k is

    a) 3   b) 0   c) -3   d) -1

155. For the function $f(x) = \frac{\log_e(1+x) + \log_e(1-x)}{x}$ to be continuous at $= 0$, the value of $f(0)$ is
    a) -1   b) 0   c) -2   d) 2

156. Let $f(x) = \begin{cases} \frac{x-4}{|x-4|} + a, & x < 4 \\ a + b, & x = 4 \\ \frac{x-4}{|x-4|} + b, & x > 4 \end{cases}$

    Then, $f(x)$ is continuous at $x = 4$, when
    a) $a = 0, b = 0$ b) $a = 1, b = 1$ c) $a = -1, b = 1$ d) $a = 1, b = -1$

157. If $f(x) \begin{cases} \frac{[x]-1}{x-1}, & x \neq 1 \\ 0, & x = 1 \end{cases}$ then at $x = 1$, $f(x)$ is

a) Continuous and differentiable

b) Differentiable but not continuous

c) Continuous but not differentiable

d) Neither continuous nor differentiable

158. If $f(x) = \begin{cases} \frac{1-\sqrt{2}\sin x}{\pi - 4x}, & \text{if } x \neq \frac{\pi}{4} \\ a, & \text{if } x = \frac{\pi}{4} \end{cases}$ is continuous at $\frac{\pi}{4}$, then a is equal to

a) 4   b) 2   c) 1   d) 1/4

159. If the function $f: R \rightarrow R$ given by $f(x) = \begin{cases} x + a, & \text{if } x \leq 1 \\ 3 - x^2, & \text{if } x > 1 \end{cases}$ is continuous at $x = 1$, thyen a is equal to

a) 4   b) 3   c) 2   d) 1

160. If $f: R \rightarrow R$ is defined by

$f(x) = \begin{cases} \frac{\cos 3x - \cos x}{x^2}, & \text{for } x \neq 0 \\ \lambda, & \text{for } x = 0 \end{cases}$ and if f is continuous at $x = 0$, then λ is equal to

a) -2   b) -4   c) -6   d) -8

161. For the function $f(x) = \begin{cases} \frac{x^3 - a^3}{x-a}, & x \neq a \\ b, & x = a \end{cases}$, if $f(x)$ is continuous at $x = a$, then b is equal to

a) a^2 b) $2a^2$ c) $3a^2$ d) $4a^2$

162. If $y = f(x) = \frac{1}{u^2 + u - 1}$ where $u = \frac{1}{x-1}$, then the function is discontinuous at $x =$

a) 1   b) 1/2   c) 2   d) -2

163. If $f(x) = \text{Min } \{\tan x, \cot x\}$, then

a) $f(x)$ is not differentiable at $x = 0, \pi/4, 5\pi/4$

b) $f(x)$ is continuous at $x = 0, \pi/2, 3\pi/2$

c) $\int_0^{\pi/2} f(x)dx = \ln \sqrt{2}$

d) $f(x)$ is periodic with period $\frac{\pi}{2}$

164. If $f(x) = \{|x| - |x - 1|\}^2$, then $f'(x)$ equals

a) 0 for all x

b) $2\{|x| - |x - 1|\}$

c) $\begin{cases} 0 \text{ for } x < 0 \text{ and for } x > 1 \\ 4(2x - 1) \text{for } 0 < x < 1 \end{cases}$

d) $\begin{cases} 0 \text{ for } x < 0 \\ 4(2x - 1) \text{for } x > 0 \end{cases}$

165. If $f(x) = (x - x_0)\phi(x)$ and $\phi(x)$ is continuous at $x = x_0$, then $f'(x_0)$ is equal to

a) $\phi'(x_0)$ b) $\phi(x_0)$ c) $x_0\phi(x_0)$ d) None of these

166. The function defined by

$f(x) = \begin{cases} \left(x^2 + e^{\frac{1}{2-x}}\right)^{-1} & x \neq 2 \\ k, & x = 2 \end{cases}$ is continuous from right at the point $x = 2$, then k is equal to

a) 0   b) $\frac{1}{4}$ c) $-\frac{1}{2}$ d) None of these

167. If $f(x) = \begin{cases} \frac{1-\sin x}{(\pi - 2x)^2} \cdot \frac{\log \sin x}{(\log 1 + \pi^2 - 4\pi x + x^2)}, & x \neq \frac{\pi}{2} \\ k, & x = \frac{\pi}{2} \end{cases}$ is continuous at $x = \pi/2$, then $k =$

a) $-\frac{1}{16}$ b) $-\frac{1}{32}$ c) $-\frac{1}{64}$ d) $-\frac{1}{28}$

168. If $f(x) = \begin{cases} \frac{\sin 5x}{x^2 + 2x}, & x \neq 0 \\ k + \frac{1}{2}, & x = 0 \end{cases}$ is continuous at $x = 0$, then the value of k is

a) 1   b) -2   c) 2   d) $\frac{1}{2}$

169. Let $f(x) = \begin{cases} x^n \sin\frac{1}{x}, & x \neq 0 \\ 0, & x = 0 \end{cases}$. Then, $f(x)$ is continuous but not differentiable at $x = 0$, if

a) $n \in (0, 1]$ b) $n \in [1, \infty)$ c) $n \in (-\infty, 0)$ d) $n = 0$

170. The function $f(x) = \begin{cases} |x - 3|, & \text{if } x \geq 1 \\ \frac{x^2}{4} - \frac{3x}{2} + \frac{13}{4}, & \text{if } x < 1 \end{cases}$ is

a) Continuous and differentiable at $x = 3$

b) Continuous at $x = 3$, but not differentiable at $x = 3$

c) continuous and differentiable everywhere

d) continuous at $x = 1$, but not differentiable at $x = 1$

171. Let $f(x) = |x|$ and $g(x) = |x^3|$, then

a) $f(x)$ and $g(x)$ Both are continuous at $x = 0$

b) $f(x)$ and $g(x)$ Both are differentiable at $x = 0$

c) $f(x)$ is differentiable but $g(x)$ is not differentiable at $x = 0$

d) $f(x)$ and $g(x)$ Both are not differentiable at $x = 0$

172. If $f(x) = \begin{cases} \frac{\sin(a+1)x + \sin x}{x}, & x < 0 \\ c, & x = 0 \\ \frac{\sqrt{x + bx^2} - \sqrt{x}}{bx\sqrt{x}}, & x > 0 \end{cases}$ is continuous at $x = 0$, then

a) $a = -\frac{3}{2}, b = 0, c = \frac{1}{2}$

b) $a = -\frac{3}{2}, b = 1, c = -\frac{1}{2}$

c) $a = -\frac{3}{2}, b \in R - \{0\}, c = \frac{1}{2}$

d) None of these

173. If $f(x) = \begin{cases} \frac{36^x - 9^x - 4^x + 1}{\sqrt{2} - \sqrt{1 + \cos x}}, & x \neq 0 \\ k, & x = 0 \end{cases}$ is continuous at $x = 0$, then k equals

a) $16\sqrt{2} \log 2 \log 3$ b) $16\sqrt{2} \ln 6$ c) $16\sqrt{2} \ln 2 \ln 3$ d) None of these

174. Let [ ] denotes the greatest integer function and $f(x) = [\tan^2 x]$. Then,

a) $\lim_{x \to 0} f(x)$ does not exist   b) $f(x)$ is continuous at $x = 0$

c) $f(x)$ is not differentiable at $x = 0$ d) $f(x) = 1$

175. Let a function $f: R \to R$, where R is the set of real numbers satisfying the equation $f(x + y) = f(x) + f(y), \forall x, y$ if $f(x)$ is continuous at $x = 0$, then

a) $f(x)$ is discontinuous, $\forall x \in R$ b) $f(x)$ is continuous, $\forall x \in R$

c) $f(x)$ is continuous for $x \in \{1, 2, 3, 4\}$ d) None of the above

176. Let $f(x) = \begin{cases} \sin x, & \text{for } x \geq 0 \\ 1 - \cos x, & \text{for } x \leq 0 \end{cases}$ and $g(x) = e^x$. Then, $(gof)'(0)$ is

a) 1   b) -1   c) 0   d) None of these

177. The function $f(x) \begin{cases} (x + 1)^{2 - \left(\frac{1}{|x|} + \frac{1}{x}\right)}, & x \neq 0 \\ 0, & x = 0 \end{cases}$ is

a) Continuous everywhere

b) Discontinuous at only one point

c) Discontinuous at exactly two points

d) None of these

178. If $f(x) = \begin{cases} \frac{\log(1+ax)-\log(1-bx)}{x}, & x \neq 0 \\ k, & x = 0 \end{cases}$ and $f(x)$ is continuous at $x = 0$, then the value of k is

   a) $a - b$ b) $a + b$ c) $\log a + \log b$ d) None of these

179. The value of $f(0)$, so that the function $f(x) = \frac{(27-2x)^{1/3}-3}{9-3(243+5x)^{1/5}}$ $(x \neq 0)$ is continuous is given by

   a) $\frac{2}{3}$ b) 6   c) 2   d) 4

180. The function $f: R/\{0\} \to R$ given by

$$f(x) = \frac{1}{x} - \frac{2}{e^{2x} - 1}$$

Can be made continuous at $x = 0$ by defining $f(0)$ as function

   a) 2   b) -1   c) 0   d) 1

181. Which one of the following is not true always?

   a) If $f(x)$ is not continuous at $x = a$, then it is not differentiable at $x = a$

   b) If $f(x)$ is continuous at $x = a$, then it is differentiable at $x = a$

   c) If $f(x)$ and $g(x)$ are differentiable at $x = a$, then $f(x) + g(x)$ is also differentiable at $x = a$

   d) If a function $f(x)$ is continuous at $x = a$, then $\lim_{x \to a} f(x)$ exists

182. The value of the derivative of $|x - 1| + |x - 3|$ at $x = 2$ is

   a) 2   b) 1   c) 0   d) -2

183. On the interval $I = [-2, 2]$, the function $f(x) = \begin{cases} (x+1)\, e^{-\left(\frac{1}{|x|}+\frac{1}{x}\right)}, & x \neq 0 \\ 0, & x = 0 \end{cases}$

   a) Is continuous for all $x \in I - \{0\}$

   b) Assumes all intermediate values from $f(-2)$ to $f(2)$

   c) Has a maximum value equal to $3/e$

   d) All the above

184. Function $f(x) = \begin{cases} x - 1, & x < 2 \\ 2x - 3, & x \geq 2 \end{cases}$ is a continuous function

   a) For $x = 2$ only   b) For all real values of x such that $x \neq 2$

   c) For all real values of x d) For all integer values of x only

185. The function $f(x) = \begin{cases} \frac{\tan x}{x}, & x \neq 0 \\ 1, & x = 0 \end{cases}$, is

   a) Continuous but not differentiable at $x = 0$ b) Discontinuous at $x = 0$

   c) Continuous and differentiable at $x = 0$ d) Not defined at $x = 0$

186. At the point $x = 1$, the function $f(x) = \begin{cases} x^3 - 1, & 1 < x < \infty \\ x - 1, & -\infty < x \leq 1 \end{cases}$

   a) Continuous and differentiable

   b) Continuous and not differentiable

   c) Discontinuous and differentiable

   d) Discontinuous and not differentiable

187. If $f(x)$ defined by $f(x) = \begin{cases} \frac{|x^2 - x|}{x^2 - x}, & x \neq 0, 1 \\ 1, & x = 0 \\ -1, & x = 1 \end{cases}$ then $f(x)$ is continuous for all

   a) x

   b) x except at $x = 0$

   c) x except at $x = 1$

   d) x except at $x = 0$ and $x = 1$

188. The value of derivative of $|x - 1| + |x - 3|$ at $x = 2$, is

   a) -2 b) 0   c) 2   d) Not defined

189. If $f(x) = \begin{cases} 1 & \text{for } x < 0 \\ 1 + \sin x & \text{for } 0 \le x \le \pi/2 \end{cases}$, then at $x = 0$, the derivative $f'(x)$ is

   a) 1   b) 0   c) Infinite   d) Does not exist

190. Let $g(x) = \frac{(x-1)^n}{\log \cos^m(x-1)}$; $0 < x < 2$, m and n are integers, $m \ne 0, n > 0$, and let p be the left hand derivative of $|x - 1|$ at $x = 1$. If $\lim\limits_{x \to 1^+} g(x) = p$, then

   a) $n = 1, m = 1$ b) $n = 1, m = -1$ c) $n = 2, m = 2$ d) $n > 2, m = n$

191. The function $f(x) = \frac{2x^2 + 7}{x^3 + 3x^2 - x - 3}$ is discontinuous for

   a) $x = 1$ only

   b) $x = 1$ and $x = -1$ only

   c) $x = 1, x = -1, x = -3$ only

   d) $x = 1, x = -1, x = -3$ and other values of x

192. If for a function $f(x)$, $f(2) = 3$, $f'(2) = 4$, then $\lim\limits_{x \to 2}[f(x)]$, where $[\cdot]$ denotes the greatest integer function, is

   a) 2   b) 3   c) 4   d) Non-existent

193. A function $f(x)$ is defined as fallows for real x,

   $f(x) = \begin{cases} 1 - x^2, & \text{for } x < 1 \\ 0, & \text{for } x = 1 \\ 1 + x^2, & \text{for } x > 1 \end{cases}$ Then,

   a) $f(x)$, is not continuous at $x = 1$

   b) $f(x)$ is continuous but not differentiable at $x = 1$

   c) $f(x)$ is both continuous and differentiable at $x = 1$

   d) None of the above

194. Let $f: R \to R$ be a function defined by $f(x) = \min\{x + 1, |x| + 1\}$. Then, which of the following is true?

   a) $f(x) \ge 1$ for all $x \in R$

   b) $f(x)$ is not differentiable at $x = 1$

   c) $f(x)$ is differentiable everywhere

   d) $f(x)$ is not differentiable at $x = 0$

195. If $f(x) = \begin{cases} mx + 1, & x \le \frac{\pi}{2} \\ \sin x + n, & x > \frac{\pi}{2} \end{cases}$ is continuous t $x = \frac{\pi}{2}$, then

   a) $m = 1, n = 0$ b) $m = \frac{n\pi}{2} + 1$ c) $n = m\frac{\pi}{2}$ d) $m = n = \frac{\pi}{2}$

196. If $f(x) = \frac{\log_e(1 + x^2 \tan x)}{\sin x^3}$, $x \ne 0$, is to be continuous at $x = 0$, then $f(0)$ must be defined as

   a) 1   b) 0   c) $\frac{1}{2}$ d) -1

197. Let $f(x) = \begin{cases} x^P \sin\frac{1}{x}, & x \ne 0 \\ 0, & x = 0 \end{cases}$ then $f(x)$ is continuous but not differentiable at $x = 0$, if

   a) $0 < p \le 1$ b) $1 \le p < \infty$ c) $-\infty < p < 0$ d) $p = 0$

198. The function f defined by

   $f(x) = \begin{cases} \frac{\sin x^2}{x}, & x \ne 0 \\ 0, & x = 0 \end{cases}$ is

   a) Continuous and derivable at $x = 0$

   b) Neither continuous nor derivable at $x = 0$

   c) Continuous but not derivable at $x = 0$

   d) None of these

199. A function f on R into itself is continuous at a point a in R, iff for each $\in > 0$, there exists, $\delta > 0$ such that

   a) $|f(x) - f(a)| < \in \Rightarrow |x - a| < \delta$ b) $|f(x) - f(a)| > \in \Rightarrow |x - a| > \delta$

   c) $|x - a| > \delta | f(x) - f(a)| > \in$ d) $|x - a| < \delta | f(x) - f(a)| < \in$

200. The function $f(x) = x - |x - x^2|$, $-1 \le x \le 1$ is continuous on the interval

   a) $[-1, 1]$ b) $(-1, 1)$ c) $[-1, 0) \cup (0, 1]$ d) $(-1, 0) \cup (0, 1)$

201. if $f(x) = a|\sin x| + b\, e^{|x|} + c\, |x|^3$ and if $f(x)$ is differentiable at $x = 0$, then

   a) $a = b = c = 0$ b) $a = 0, b = 0; c \in R$ c) $b = c = 0, a \in R$ d) $c = 0, a = 0, b \in R$

202. Let $f(x)$ be defined on R such that $f(1) = 2, f(2) = 8$ and $f(u + v) = f(u) + kuv - 2v^2$ for all $u, v \in R$ (k is a fixed constant). Then,

    a) $f'(x) = 8x$ b) $f(x) = 8x$ c) $f'(x) = x$ d) None of these

203. If $f(x) = \sin^{-1}\left(\frac{2x}{1+x^2}\right)$, then $f(x)$ is differentiable on

    a) $[-1, 1]$ b) $R - \{-1, 1\}$ c) $R - (-1, 1)$ d) None of these

204. Define f on R into itself by

$$f(x) = \begin{cases} x \sin\frac{1}{x}, & \text{when } x \neq 0 \\ 0, & \text{when } x = 0 \end{cases}, \text{ then}$$

    a) f is continuous at 0 but not differentiable at 0   b) f is both continuous and differentiable at 0

    c) f is differentiable but not continuous at 0   d) None of the above

205. The set of points where the function $f(x) = |x - 1|e^x$ is differentiable, is

    a) R b) $R - \{1\}$ c) $R - \{-1\}$ d) $R - \{0\}$

206. Let $f(x + y) = f(x)f(y)$ and $f(x) = 1 + xg(x)G(x)$, where $\lim\limits_{x \to 0} g(x) = a$ and $\lim\limits_{x \to 0} G(x) = b$. Then $f'(x)$ is equal to

    a) $1 + ab$ b) ab c) a/b d) None of these

CONTINUITY AND DIFFERENTIABILITY

: ANSWER KEY :

1)	b	2)	d	3)	b	4)	c	109)	b	110)	a	111)	a	112)	c
5)	b	6)	b	7)	b	8)	c	113)	b	114)	c	115)	c	116)	c
9)	a	10)	d	11)	d	12)	a	117)	c	118)	b	119)	b	120)	c
13)	a	14)	b	15)	d	16)	a	121)	a	122)	c	123)	b	124)	d
17)	a	18)	b	19)	c	20)	b	125)	c	126)	b	127)	a	128)	b
21)	b	22)	c	23)	d	24)	b	129)	d	130)	b	131)	b	132)	a
25)	a	26)	d	27)	c	28)	b	133)	a	134)	d	135)	a	136)	d
29)	d	30)	b	31)	b	32)	b	137)	a	138)	b	139)	d	140)	a
33)	b	34)	c	35)	d	36)	c	141)	c	142)	d	143)	a	144)	a
37)	b	38)	a	39)	a	40)	a	145)	a	146)	d	147)	a	148)	b
41)	c	42)	b	43)	b	44)	c	149)	c	150)	b	151)	a	152)	c
45)	d	46)	c	47)	d	48)	b	153)	b	154)	a	155)	b	156)	d
49)	c	50)	d	51)	c	52)	a	157)	d	158)	d	159)	d	160)	b
53)	c	54)	b	55)	c	56)	d	161)	c	162)	a	163)	a	164)	c
57)	c	58)	d	59)	b	60)	c	165)	b	166)	b	167)	c	168)	c
61)	a	62)	a	63)	a	64)	b	169)	a	170)	b	171)	a	172)	c
65)	d	66)	a	67)	c	68)	d	173)	c	174)	b	175)	b	176)	c
69)	d	70)	a	71)	a	72)	b	177)	b	178)	b	179)	c	180)	d
73)	b	74)	c	75)	d	76)	b	181)	b	182)	c	183)	d	184)	c
77)	b	78)	c	79)	b	80)	b	185)	c	186)	b	187)	d	188)	b
81)	d	82)	c	83)	a	84)	b	189)	d	190)	c	191)	c	192)	c
85)	c	86)	c	87)	c	88)	b	193)	a	194)	c	195)	c	196)	a
89)	c	90)	a	91)	b	92)	d	197)	a	198)	a	199)	a	200)	a
93)	b	94)	c	95)	b	96)	c	201)	b	202)	a	203)	b	204)	a
97)	d	98)	d	99)	d	100)	d	205)	b	206)	d				
101)	a	102)	b	103)	c	104)	c								
105)	a	106)	a	107)	d	108)	a								

CONTINUITY AND DIFFERENTIABILITY

1 **(b)**

We have,

$-\pi/4 < x < \pi/4$

$\Rightarrow -1 < \tan x < 1 \Rightarrow 0 \le \tan^2 x < 1 \Rightarrow [\tan^2 x]$
$\quad = 0$

$\therefore f(x) = [\tan^2 x] = 0$ for all $x \in (-\pi/4, \pi/4)$

Thus, $f(x)$ is a constant function on $\in (-\pi/4, \pi/4)$

So, it is continuous on $\in (-\pi/4, \pi/4)$ and $f'(x) = 0$ for all $x \in (-\pi/4, \pi/4)$

2 **(d)**

Since, $f(x)$ is continuous at $x = 0$

$\therefore \quad \lim_{x \to 0} f(x) = f(0)$

$\Rightarrow \lim_{x \to 0} \frac{-e^x + 2^x}{x} = f(0)$

$\Rightarrow \lim_{x \to 0} \frac{-e^x + 2^x \log 2}{1} = f(0)$ [by L 'Hospital's rule]

$\Rightarrow f(0) = -1 + \log 2$

3 **(b)**

Since $f(x)$ is an even function

$\therefore f(-x) = f(x)$ for all x

$\Rightarrow -f'(-x) = f'(x)$ for all x

$\Rightarrow f'(-x) = -f'(x)$ for all x

$\Rightarrow f'(x)$ is an odd function

4 **(c)**

We have,

$f(x) = \begin{cases} [\cos \pi x], & x < 1 \\ |x - 2|, & 1 \le x < 2 \end{cases}$

$\Rightarrow f(x) = \begin{cases} 2 - x, & 1 \le x < 2 \\ -1, & 1/2 < x < 1 \\ 0, & 0 < x \le 1/2 \\ 1, & x = 0 \\ 0, & -1/2 \le x < 0 \\ -1, & -3/2 < x < -1/2 \end{cases}$

It is evident from the definition that $f(x)$ is discontinuous at $x = 1/2$

5 **(b)**

We have,

$\lim_{x \to 2^-} f(x) = \lim_{h \to 0} f(2 - h)$

$\quad = \lim_{h \to 0} \frac{|-2 - h + 2|}{\tan^{-1}(-2 - h + 2)}$

$\Rightarrow \lim_{x \to 2^-} f(x) = \lim_{h \to 0} \frac{h}{\tan^{-1}(-h)} = \lim_{h \to 0} \frac{-h}{\tan^{-1} h} = -1$

and,

$\lim_{x \to -2^+} f(x) = \lim_{h \to 0} f(-2 + h)$

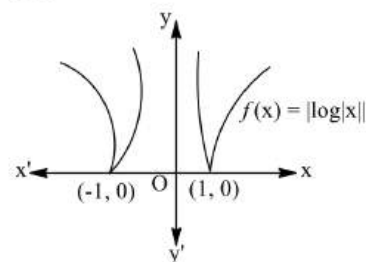$\quad = \lim_{h \to 0} \frac{|-2 + h + 2|}{\tan^{-1}(-2 + h + 2)}$

$\Rightarrow \lim_{x \to -2^+} f(x) = \lim_{h \to 0} \frac{h}{\tan^{-1} h} = 1$

$\therefore \lim_{x \to -2^-} f(x) \ne \lim_{\to -2^+} f(x)$

So, $f(x)$ is neither continuous nor differentiable at $x = -2$

6 **(b)**

From the graph of $f(x) = |\log|x||$ it is clear that $f(x)$ is everywhere continuous but not differentiable at $x = \pm 1$, due to sharp edge

7 **(b)**

We have,

$\lim_{x \to a} \frac{x f(a) - a f(x)}{x - a}$

$= \lim_{x \to a} \frac{x f(a) - a f(a) - a(f(x) - f(a))}{x - a}$

$\Rightarrow \lim_{x \to a} \frac{x f(a) - a f(x)}{x - a}$

$\quad = \lim_{x \to a} \frac{f(a)(x - a)}{x - a}$

$\quad - a \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$

$\Rightarrow \lim\limits_{x \to a} \dfrac{x\,f(a) - a\,f(x)}{x - a} = f(a) - a\,f'(a) = 4 - 2a$

8 **(c)**

Given, $f(x) = x\left(\sqrt{x} + \sqrt{x+1}\right)$. At $x = 0$ LHL of $\sqrt{x}$ is not defined, therefore it is not continuous at $x = 0$

Hence, it is not differentiable at $x = 0$

9 **(a)**

Here, $f'(x) = \begin{cases} 2ax, & b \neq 0, x \leq 1 \\ 2bx + a, & x > 1 \end{cases}$

Since, $f(X)$ is continuous at $x = 1$

$\therefore \quad \lim\limits_{h \to 0} f(x) = \lim\limits_{h \to 1^+} f(x)$

$\Rightarrow \quad a + b = b + a + c \Rightarrow c = 0$

Also, $f(x)$ is differentiable at $x = 1$

$\therefore$ (LHD at $x = 1$)=(RHD at $x = 1$)

$\Rightarrow \quad 2a = 2b(1) + a \Rightarrow a = 2b$

10 **(d)**

We have,

$\lim\limits_{x \to 1^-} f(x) = \lim\limits_{x \to 1^-} \left\{ \dfrac{x^2}{4} - \dfrac{3x}{4} + \dfrac{13}{4} \right\} = \dfrac{1}{4} - \dfrac{3}{2} + \dfrac{13}{4}$

$\qquad\qquad = 2$

$\lim\limits_{x \to 1^+} f(x) = \lim\limits_{x \to 1} |x - 3| = 2$

and, $f(1) = |1 - 3| = 2$

$\therefore \lim\limits_{x \to 1^-} f(x) = f(1) = \lim\limits_{x \to 1^+} f(x)$

So, $f(x)$ is continuous at $x = 1$

We have,

$\lim\limits_{x \to 3^-} f(x) = \lim\limits_{x \to 3} |x - 3| = 0,\ \lim\limits_{x \to 3^+} f(x)$

$\qquad\qquad = \lim\limits_{x \to 3} |x - 3| = 0$

and, $f(3) = 0$

$\therefore \lim\limits_{x \to 3^-} f(x) = \lim\limits_{x \to 3^+} f(x) = f(3)$

So, $f(x)$ is continuous at $x = 3$

Now,

(LHD at $x = 1$)

$= \left\{ \dfrac{d}{dx}\left(\dfrac{x^2}{4} - \dfrac{3x}{2} + \dfrac{13}{4}\right) \right\}_{x=1} = \left\{ \dfrac{x}{2} - \dfrac{3}{2} \right\}_{x=1} = \dfrac{1}{2} - \dfrac{3}{2}$

$\qquad\qquad = -1$

(RHD at $x = 1$) $= \left\{ \dfrac{d}{dx}\left(-(x - 3)\right) \right\}_{x=1} = -1$

$\therefore$ (LHD at $x = 1$) = (RHD at $x = 1$)

So, $f(x)$ is differentiable at $x = 1$

11 **(d)**

$f(x) = \begin{cases} \dfrac{2\sin x - \sin 2x}{2x \cos x}, & \text{if } x \neq 0, \\ a, & \text{if } x = 0 \end{cases}$

Now, $\lim\limits_{x \to 0} f(x) = \lim\limits_{x \to 0} \dfrac{2\sin x - \sin 2x}{2x \cos x} \quad \left(\dfrac{0}{0}\ \text{form}\right)$

$= \lim\limits_{x \to 0} \dfrac{2\cos x - 2\cos 2x}{2(\cos x - x \sin x)}$

$= \lim\limits_{x \to 0} \dfrac{2 - 2}{2(1 - 0)} = 0$
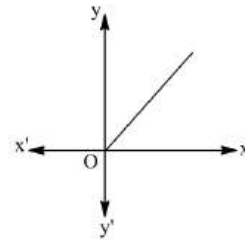
Since, $f(x)$ is continuous at $x = 0$

$\therefore \quad f(0) = \lim\limits_{x \to 0} f(x)$

$\Rightarrow a = 0$

12 **(a)**

Given, $f(x) = x + |x|$

$\therefore \quad f(x) = \begin{cases} 2x, & x \geq 0 \\ 0, & x < 0 \end{cases}$

It is clear from the graph of $f(x)$ is continuous for every value of x

Alternate

Since, x and $|x|$ is continuous for every value of x, so their sum is also continous for every value of x

13 **(a)**

Since $f(x)$ is continuous at $x = 0$

$\therefore \lim\limits_{x \to 0^-} f(x) = f(0) = \lim\limits_{x \to 0^+} f(x)$

$\Rightarrow \lim\limits_{x \to 0}\{1 + |\sin x|\}^{\frac{a}{|\sin x|}} = b = \lim\limits_{x \to 0} e^{\frac{\tan 2x}{\tan 3x}}$

$\Rightarrow e^a = b = e^{2/3} \Rightarrow a = \dfrac{2}{3}$ and $a = \log_e b$

14 **(b)**

We have,

$f(x) = \begin{cases} x^2 + \dfrac{(x^2/1 + x^2)}{1 - (1/1 + x^2)} = x^2 + 1, & x \neq 0 \\ 0, & x = 0 \end{cases}$

Clearly, $\lim\limits_{x \to 0^-} f(x) = \lim\limits_{x \to 0^+} f(x) = 1 \neq f(0)$

So, $f(x)$ is discontinuous at $x = 0$

15 **(d)**

LHD $= \lim\limits_{h \to 0} \dfrac{f(0 - h) - f(0)}{-h}$

$= \lim\limits_{h \to 0} \dfrac{1 - 1}{-h} = 0$

RHD $= \lim\limits_{h \to 0} \dfrac{f(0 + h) - f(0)}{h}$

$= \lim\limits_{h \to 0} \dfrac{1 + \sin(0 + h) - 1}{h} = \lim\limits_{h \to 0} \dfrac{\sin h}{h} = 1$

$\Rightarrow$ LHD$\neq$RHD

16 **(a)**

Given, $f(x) = x - |x - x^2|$

At $x = 1$, $f(1) = 1 - |1 - 1| = 1$

$\lim\limits_{x \to 1^-} f(x) = \lim\limits_{h \to 0}[(1 - h) - |(1 - h)-(1 - h)^2|]$

$= \lim\limits_{h \to 0} [(1 - h) - |h - h^2|] = 1$

$\lim\limits_{x \to 1^+} f(x) = \lim\limits_{h \to 0}[(1 + h) - |(1 + h) - (1 + h)^2|]$

$$= \lim_{h \to 0} [1 + h - |-h^2 - h|] = 1$$

$$\therefore \lim_{x \to 1^{-1}} f(x) = \lim_{x \to 1^+} = f(1)$$

17 (a)

We have,

$f(x + y + z) = f(x)f(y)f(z)$ for all x, y, z ...(i)

$\Rightarrow f(0) = f(0)f(0)f(0)$ [Putting $x = y = z = 0$]

$\Rightarrow f(0)\{1 - f(0)^2\} = 0$

$\Rightarrow f(0) = 1$ [$\because f(0) = 0 \Rightarrow f(x) = 0$ for all x]

Putting $z = 0$ and $y = 2$ in (i), we get

$f(x + 2) = f(x)f(2)f(0)$

$\Rightarrow f(x + 2) = 4f(x)$ for all x

$\Rightarrow f'(2) = 4f'(0)$ [Putting $x = 0$]

$\Rightarrow f'(2) = 4 \times 3 = 12$

18 (b)

For $x > 1$, we have

$$f(x) = |\log|x|| = \log x \quad \Rightarrow \quad f'(x) = \frac{1}{x}$$

For $x < -1$, we have

$$f(x) = |\log|x|| = \log(-x) \quad \Rightarrow \quad f'(x) = \frac{1}{x}$$

For $0 < x < 1$, we have

$$f(x) = |\log|x|| = -\log x \quad \Rightarrow \quad f'(x) = \frac{-1}{x}$$

For $-1 < x < 0$, we have

$$f(x) = -\log(-x) \quad \Rightarrow \quad f'(x) = -\frac{1}{x}$$

Hence, $f'(x) = \begin{cases} \frac{1}{x}, & |x| > 1 \\ -\frac{1}{x}, & |x| < 1 \end{cases}$

19 (c)

Since, $\lim_{x \to 0} f(x) = f(0)$

$\Rightarrow \quad \lim_{x \to 0} \dfrac{1 - \cos x}{x^2} = k$

$\Rightarrow \quad \lim_{x \to 0} \dfrac{-(-\sin x)}{2x} = k$ [using L'Hospital's rule]

$\Rightarrow \quad \dfrac{1}{2} \lim_{x \to 0} \dfrac{\sin x}{x} = k \quad \Rightarrow \quad k = \dfrac{1}{2}$

20 (b)

Given, $f(X) = |x - 1| + |x - 2|$

$= \begin{cases} x - 1 + x - 2, & x \geq 2 \\ x - 1 + 2 - x, & 1 \leq x < 2 \\ 1 - x + 2 - x, & x < 1 \end{cases}$

$= \begin{cases} 2x - 3, & x \geq 2 \\ 1, & 1 \leq x < 2 \\ 3 - 2x, & x < 1 \end{cases}$

$f'(x) = \begin{cases} 2, & x > 2 \\ 0, & 1 < x < 2 \\ -1, & x < 1 \end{cases}$

Hence, except $x = 1$ and $x = 2$, $f(x)$ is differentiable everywhere in R

21 (b)

Clearly, $f(x)$ is differentiable for all non-zero values of x. For $x \neq 0$, we have

$$f'(x) = \frac{x\, e^{-x^2}}{\sqrt{1 - e^{-x^2}}}$$

Now,

(LHD at $x = 0$) $= \lim_{x \to 0^-} \dfrac{f(x) - f(0)}{x - 0}$

$= \lim_{h \to 0} \dfrac{f(0 - h) - f(0)}{x - 0}$

$\Rightarrow$ (LHD at $x = 0$) $= \lim_{h \to 0} \dfrac{\sqrt{1 - e^{-h^2}}}{-h}$

$= \lim_{h \to 0} -\dfrac{\sqrt{1 - e^{-h^2}}}{h}$

$\Rightarrow$ (LHD at $x = 0$) $= -\lim_{h \to 0} \sqrt{\dfrac{e^{h^2} - 1}{h^2}} \times \dfrac{1}{\sqrt{e^{h^2}}} = -1$

and, (RHD at $x = 0$) $= \lim_{x \to 0^+} \dfrac{f(x) - f(0)}{x - 0} =$

$\lim_{h \to 0} \dfrac{\sqrt{1 - e^{-h^2}} - 0}{h}$

$\Rightarrow$ (RHD at $x = 0$) $= \lim_{h \to 0} \sqrt{\dfrac{e^{h^2} - 1}{h^2}} \times \dfrac{1}{\sqrt{e^{h^2}}} = 1$

So, $f(x)$ is not differentiable at $x = 0$

Hence, the set of points of differentiability of $f(x)$ is $(-\infty, 0) \cup (0, \infty)$

22 (c)

Since $f(x)$ is continuous at $x = 0$

$\therefore f(0) = \lim_{x \to 0} x \sin\left(\dfrac{1}{x}\right) = 0$

23 (d)

For $f(x)$ to be continuous everywhere, we must have,

$f(0) = \lim_{x \to 0} f(x)$

$\Rightarrow f(0) = \lim_{x \to 0} \dfrac{2 - (256 - 7x)^{1/8}}{(5x + 32)^{1/5} - 2}$ $\left[\text{Form } \dfrac{0}{0}\right]$

$\Rightarrow f(0) = \lim_{x \to 0} \dfrac{\frac{7}{8}(256 - 7x)^{-\frac{7}{8}}}{(5x + 32)^{-4/5}} = \dfrac{7}{8} \times \dfrac{2^{-7}}{2^{-4}} = \dfrac{7}{64}$

24 (b)

We have,

$f(x) = |x|^3 = \begin{cases} x^3, & x \geq 0 \\ -x^3, & x < 0 \end{cases}$

$\therefore$ (LHD at $x = 0$) $= \lim_{x \to 0^-} \dfrac{f(x) - f(0)}{x - 0} = \lim_{x \to 0} -\dfrac{x^3}{x}$

$= 0$

and,

$\therefore$ (RHD at $x = 0$) $= \lim_{x \to 0^+} \dfrac{f(x) - f(0)}{x - 0} = \lim_{x \to 0} \dfrac{x^3}{x}$

$= 0$

Clearly, (LHD at $x = 0$) = (RHD at $x = 0$)

Hence, $f(x)$ is differentiable at $x = 0$ and its derivative at $x = 0$ is 0

25 (a)

$$\lim_{x \to 0} f(x) = \lim_{x \to 0} \left(\frac{4^x - 1}{x}\right)^3 \times \frac{\left(\frac{x}{a}\right)}{\sin\left(\frac{x}{a}\right)} \cdot \frac{ax^2}{\log\left(1 + \frac{1}{3}x^2\right)}$$

$$= (\log 4)^3 \cdot 1 \cdot a \lim_{x \to 0} \left(\frac{x^2}{\frac{1}{3}x^2 - \frac{1}{18}x^4 + \dots}\right)$$

$$= 3a \, (\log 4)^3$$

$\because \quad \lim_{x \to 0} f(x) = f(0)$

$\Rightarrow \quad 3a \, (\log 4)^3 = 9(\log 4)^3$

$\Rightarrow \quad a = 3$

26 (d)

We have,

$f(x) = |[x]x|$ for $-1 < x \le 2$

$$\Rightarrow f(x) = \begin{cases} -x, & -1 < x < 0 \\ 0, & 0 \le x < 1 \\ x, & 1 \le x < 2 \\ 2x, & x = 2 \end{cases}$$

It is evident from the graph of this function that it is continuous but not differentiable at $x = 0$. Also, it is discontinuous at $x = 1$ and non-differentiable at $x = 2$

27 (c)

Given, $f(x) = [x^3 - 3]$

Let $g(x) = x^3 - x$ it is in increasing function

$\therefore \quad g(1) = 1 - 3 = -2$

and $g(2) = 8 - 3 = 5$

Here, $f(x)$ is discontinuous at six points

28 (b)

Given, $y = \cos^{-1} \cos(x - 1), \quad x > 0$

$\Rightarrow \quad y = x - 1, \quad 0 \le x - 1 \le \pi$

$\therefore \quad y = x - 1, \quad 1 \le x \le \pi + 1$

At $x = \frac{5\pi}{4} \in [1, \pi + 1]$

$\Rightarrow \quad \frac{dy}{dx} = 1 \quad \Rightarrow \quad \left(\frac{dy}{dx}\right)_{x = \frac{5\pi}{4}} = 1$

29 (d)

We have,

$$f'(x) = \lim_{h \to 0} \frac{f(x + h) - f(x)}{h}$$

$$\Rightarrow f'(x) = \lim_{h \to 0} \frac{f(x) + f(h) - f(x)}{h} \qquad [\because f(x + y)$$

$$= f(x) + f(y)]$$

$$\Rightarrow f'(x) = \lim_{h \to 0} \frac{f(h)}{h} = \lim_{h \to 0} \frac{h^2 g(h)}{h}$$

$$\Rightarrow f'(x) = 0 \times g(0) = 0 \quad \begin{bmatrix} \because g \text{ is continuous} \\ \therefore \lim_{h \to 0} g(h) = g(0) \end{bmatrix}$$

30 (b)

Using Heine's definition of continuity, it can be shown that $f(x)$ is everywhere discontinuous

31 (b)

For $x \ne -1$, we have

$f(x) = 1 - 2x + 3x^2 - 4x^3 + \dots \infty$

$\Rightarrow f(x) = (1 + x)^{-2} = \frac{1}{(1 + x)^2}$

Thus, we have

$$f(x) = \begin{cases} \frac{1}{(1 + x)^2}, & x \ne -1 \\ 1, & x = -1 \end{cases}$$

We have, $\lim_{x \to -1^-} f(x) \to \infty$ and $\lim_{x \to -1^-} f(x) \to \infty$

So, $f(x)$ is not continuous at $x = -1$

Consequently, it is not differentiable there at

32 (b)

At $x = a$,

LHL $= \lim_{x \to a^-} f(x) = \lim_{x \to a} 2a - x = a$

And RHL $= \lim_{x \to a^+} f(x) = \lim_{x \to a} 3x - 2a = a$

And $f(a) = 3(a) - 2a = a$

$\therefore$ LHL=RHL$= f(a)$

Hence, it is continuous at $x = a$

Again, at $x = a$

LHD $= \lim_{h \to 0} \frac{f(a - h) - f(a)}{-h}$

$$= \lim_{h \to 0} \frac{2a - (a - h) - a}{-h} = -1$$

and RHD $= \lim_{h \to 0} \frac{f(a + h) - f(a)}{h}$

$$= \lim_{h \to 0} \frac{3(a + h) - 2a - a}{h} = 3$$

$\therefore$ LHD$\ne$RHD

Hence, it is not differentiable at $x = a$

33 (b)

We have,

$$f'(x) = \lim_{h \to 0} \frac{f(x + h) - f(x)}{h}$$

$$\Rightarrow f'(x) = \lim_{h \to 0} \frac{f(x)f(h) - f(x)}{h}$$

$$\Rightarrow f'(x) = f(x) \lim_{h \to 0} \frac{f(h) - 1}{h}$$

$$\Rightarrow f'(x) = f(x) \lim_{h \to 0} \frac{1 + (\sin 2h)g(h) - 1}{h}$$

$$\Rightarrow f'(x) = f(x) \lim_{h \to 0} \frac{\sin 2h}{h} \times \lim_{h \to 0} g(h)$$

$$= 2f(x)g(0)$$

34 **(c)**

If $-1 \leq x \leq 1$, then $0 \leq x \sin \pi x \leq 1/2$

$\therefore f(x) = [x \sin \pi x] = 0$, for $-1 \leq x \leq 1$

If $1 < x < 1 + h$, where h is a small positive real number, then

$\pi < \pi x < \pi + \pi h \Rightarrow -1 < \sin \pi x < 0 \Rightarrow -1 < x \sin \pi x < 0$

$\therefore f(x) = [x \sin \pi x] = -1$ in the right neighbourhood of $x = 1$

Thus, $f(x)$ is constant and equal to zero in $[-1, 1]$ and so $f(x)$ is differentiable and hence continuous on $(-1, 1)$

At $x = 1$, $f(x)$ is discontinuous because

$\Rightarrow \lim_{x \to 1^-} f(x) = 0$ and $\lim_{x \to 1^+} f(x) = -1$

Hence, $f(x)$ is not differentiable at $x = 1$

35 **(d)**

We have,

(LHD at $x = 0$) $= \left\{ \dfrac{d}{dx}(1) \right\}_{x=0} = 0$

(RHD at $x = 0$) $= \left\{ \dfrac{d}{dx}(1 + \sin x) \right\}_{x=0} = \cos 0 = 1$

Hence, $f'(x)$ at $x = 0$ does not exist

36 **(c)**

Here, $f'(x) = \begin{cases} 2bx + a, & x \geq -1 \\ 2a, & x < -1 \end{cases}$

Given, $f'(x)$ is continuous everywhere

$\therefore \quad \lim_{x \to -1^+} f(x) = \lim_{x \to -1^-} f(x)$

$\Rightarrow -2b + a = -2a$

$\Rightarrow 3a = 2b$

$\Rightarrow a = 2, \qquad b = 3$

or $a = -2, \; b = -3$

37 **(b)**

We have,

$\lim_{x \to 0} \dfrac{f(x) - f(0)}{x - 0} = \lim_{x \to 0} \dfrac{\log \cos x}{\log(1 + x^2)}$

$\Rightarrow \lim_{x \to 0} \dfrac{f(x) - f(0)}{x - 0}$

$\qquad = \lim_{x \to 0} \dfrac{\log(1 - 1 + \cos x)}{\log(1 + x^2)}$

$\qquad \cdot \dfrac{1 - \cos x}{1 - \cos x}$

$\Rightarrow \lim_{x \to 0} \dfrac{f(x) - f(0)}{x - 0}$

$\qquad = \lim_{x \to 0} \dfrac{\log\{1 - (1 - \cos x)\}}{1 - \cos x}$

$\qquad \cdot \dfrac{1 - \cos x}{\log(1 + x^2)}$

$\Rightarrow \lim_{x \to 0} \dfrac{f(x) - f(0)}{x - 0}$

$\qquad = -\lim_{x \to 0} \log \dfrac{[1 - (1 - \cos x)]}{-(1 - \cos x)}$

$\qquad \times \dfrac{2 \sin^2 \frac{x}{2}}{4 \left(\frac{x}{2} \right)^2} \times \dfrac{x^2}{\log(1 + x^2)}$

$\Rightarrow \lim_{x \to 0} \dfrac{f(x) - f(0)}{x - 0} = -\dfrac{1}{2}$

Hence, $f(x)$ is differentiable and hence continuous at $x = 0$

38 **(a)**

Since $f(x)$ is continuous at $x = 1$. Therefore,

$\lim_{x \to 1^-} f(x) = \lim_{x \to 1^+} f(x) \Rightarrow A - B = 3 \Rightarrow A = 3 + B$ …(i)

If $f(x)$ is continuous at $x = 2$, then

$\lim_{x \to 2^-} f(x) = \lim_{x \to 2^+} f(x) \Rightarrow 6 = 4B - A$ …(ii)

Solving (i) and (ii) we get $B = 3$

As $f(x)$ is not continuous at $x = 2$. Therefore, $B \neq 3$

Hence, $A = 3 + B$ and $B \neq 3$

39 **(a)**

We have,

$f(x) = \begin{cases} x - 4, & x \geq 4 \\ -(x - 4), & 1 \leq x < 4 \\ (x^3/2) - x^2 + 3x + (1/2), & x < 1 \end{cases}$

Clearly, $f(x)$ is continuous for all x but it is not differentiable at $x = 1$ and $x = 4$

40 **(a)**

It is given that $f(x)$ is continuous at $x = 1$

$\therefore \lim_{x \to 1^-} f(x) = \lim_{x \to 1^+} f(x) = f(1)$

$\Rightarrow \lim_{x \to 1^-} a[x + 1] + b[x - 1]$

$\qquad = \lim_{x \to 1^+} a[x + 1] + b[x - 1]$

$\Rightarrow a - b = 2a + 0 \times b$

$\Rightarrow a + b = 0$

41 **(c)**

$\lim_{x \to 0^+} f(x) = \lim_{x \to 0^+} \lambda[x] = 0$

$\lim_{x \to 0^-} f(x) = \lim_{x \to 0^-} 5^{1/x} = 0$

And $f(0) = \lambda[0] = 0$

$\therefore f$ is continuous only whatever λ may be

42 **(b)**

We have,

$y(x) = f(e^x) \, e^{f(x)}$

$\Rightarrow y'(x) = f'(e^x) \cdot e^x \cdot e^{f(x)} + f(e^x) \, e^{f(x)} f'(x)$

$\Rightarrow y'(0) = f'(1)e^{f(0)} + f(1)e^{f(0)}f'(0)$

$\Rightarrow y'(0) = 2 \qquad [\because f(0) = f(1) = 0, f'(1) = 2]$

43 **(b)**

Since $f(x)$ is differentiable at $x = 1$. Therefore,

$$\lim_{x\to 1^-}\frac{f(x)-f(1)}{x-1}=\lim_{x\to 1^+}\frac{f(x)-f(1)}{x-1}$$

$$\Rightarrow \lim_{h\to 0}\frac{f(1-h)-f(1)}{-h}=\lim_{h\to 0}\frac{f(1+h)-f(1)}{h}$$

$$\Rightarrow \lim_{h\to 0}\frac{a(1-h)^2-b-1}{-h}=\lim_{h\to 0}\frac{\frac{1}{|1+h|}-1}{h}$$

$$\Rightarrow \lim_{h\to 0}\frac{(a-b-1)-2ah+ah^2}{-h}=\lim_{h\to 0}\frac{-h}{h(1+h)}$$

$$\Rightarrow \lim_{h\to 0}\frac{-(a-b-1)-2ah-ah^2}{h}=-1$$

$$\Rightarrow -(a-b-1)=0 \text{ and so } \lim_{h\to 0}\frac{2ah-ah^2}{h}=-1$$

$$\Rightarrow a-b-1=0 \text{ and } 2a=-1 \Rightarrow a=-\frac{1}{2}, b=-\frac{3}{2}$$

44 (c)

We have,

$f(x)=\frac{\sin 4\pi[x]}{1+[x]^2}=0$ for all x [$\because$

$4\pi[x]$ is a multiple of π]

$\Rightarrow f'(x)=0$ for all x
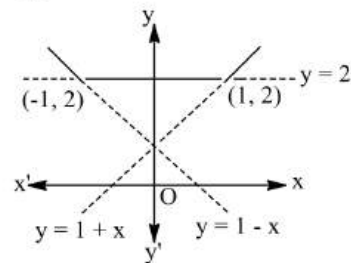
45 (d)

We have,

$$\lim_{x\to 0}f(x)=\lim_{x\to 0}\sin\frac{1}{x}$$

$\Rightarrow \lim_{x\to 0}f(x)=$ An oscillating number which

oscillates between -1 and 1

Hence, $\lim_{x\to 0}f(x)$ does not exist

Consequently, $f(x)$ cannot be continuous at $x=0$ for any value of k

46 (c)

It is clear from the graph that $f(x)$ is continuous everywhere and also differentiable everywhere except $\{-1,1\}$ due to sharp edge

47 (d)

We have,

$\log\left(\frac{x}{y}\right)=\log x-\log y$ and $\log(e)=1$

$\therefore f(x)=\log x$

Clearly, $f(x)$ is unbounded because $f(x)\to -\infty$ as $x\to 0$ and $f(x)\to +\infty$ as $x\to \infty$

We have,

$f\left(\frac{1}{x}\right)=\log\left(\frac{1}{x}\right)=-\log x$

As $x\to 0, f\left(\frac{1}{x}\right)\to \infty$

Also,

$$\lim_{x\to 0}xf(x)=\lim_{x\to 0}x\log x=\lim_{x\to 0}\frac{\log x}{1/x}$$

$$\Rightarrow \lim_{x\to 0}xf(x)=\lim_{x\to 0}\frac{1/x}{-1/x^2}=-\lim_{x\to 0}x=0$$

49 (c)

Since $g(x)$ is the inverse of $f(x)$. Therefore,

$fog(x)=x$, for all x

$\Rightarrow \frac{d}{dx}\{fog(x)\}=1$, for all x

$\Rightarrow f'(g(x))g'(x)=1$, for all x

$\Rightarrow \frac{1}{1+\{g(x)\}^3}\times g'(x)=1$ for all x $\quad\left[\because f'(x)=\frac{1}{1+x^3}\right]$

$\Rightarrow g'(x)=1+\{g(x)\}^3$, for all x

50 (d)

We have,

$f(x)=|x^2-4x+3|$

$\Rightarrow f(x)=\begin{cases} x^2-4x+3, & \text{if } x^2-4x+3\geq 0\\ -(x^2-4x+3), & \text{if } x^2-4x+3<0\end{cases}$

$\Rightarrow f(x)=\begin{cases} x^2-4x+3, & \text{if } x\leq 1 \text{ or } x\geq 3\\ -x^2+4x-3, & \text{if } 1<x<3\end{cases}$

Clearly, $f(x)$ is everywhere continuous

Now,

$(\text{LHD at } x=1)=\left(\frac{d}{dx}(x^2-4x+3)\right)_{\text{at } x=1}$

$\Rightarrow (\text{LHD at } x=1)=(2x-4)_{\text{at } x=1}=-2$

and,

$(\text{RHD at } x=1)=\left(\frac{d}{dx}(-x^2+4x-3)\right)_{\text{at } x=1}$

$\Rightarrow (\text{RHD at } x=1)=(-2x+4)_{\text{at } x=1}=2$

Clearly, $(\text{LHD at } x=1)\neq (\text{RHD at } x=1)$

So, $f(x)$ is not differentiable at $x=1$

Similarly, it can be checked that $f(x)$ is not differentiable at $x=3$ also

<u>ALITER</u> We have,

$f(x)=|x^2-4x+3|=|x-1||x-3|$

Since, $|x-1|$ and $|x-3|$ are not differentiable at 1 and 3 respectively

Therefore, $f(x)$ is not differentiable at $x=1$ and $x=3$

51 (c)

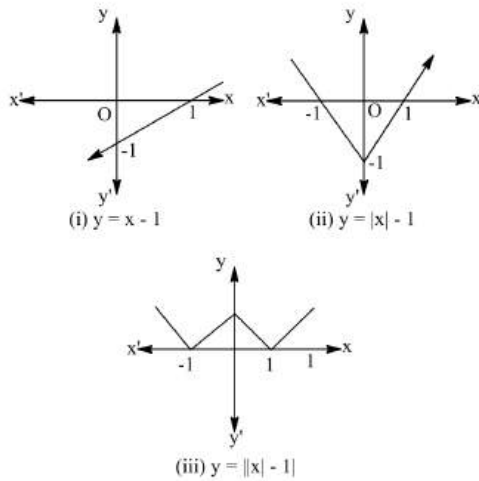The point of discontinuity of $f(x)$ are those points where $\tan x$ is infinite.

ie, $\tan x=\tan\infty$

$\Rightarrow \quad x=(2n+1)\frac{\pi}{2}, \quad n\in I$

52 (a)

Using graphical transformation

(i) y = x - 1

(ii) y = |x| - 1

(iii) y = ||x| - 1|

As, we know the function is not differentiable at 6 sharp edges and in figure (iii) $y = \left| |x| - 1 \right|$ we have 3 sharp edges at $x = -1, 0, 1$

$\therefore f(x)$ is not differentiable at $\{0, \pm 1\}$

53 (c)

$\lim\limits_{x \to 0^-} f(x) = \lim\limits_{h \to 0} 2(0 - h) = 0$

And $\lim\limits_{x \to 0^+} f(x) = \lim\limits_{h \to 0} 2(0 + h) + 1 = 1$

$\because \lim\limits_{x \to 0^-} f(x) \neq \lim\limits_{x \to 1^+} f(x)$

$\therefore f(x)$ is discontinuous at $x = 0$

54 (b)

Draw a rough sketch of $y = f(x)$ and observe its properties

55 (c)

$\lim\limits_{x \to \pi} \dfrac{(1 + \cos x) - \sin x}{(1 + \cos x) + \sin x}$

$= \lim\limits_{x \to \pi} \dfrac{2\cos^2 x/2 - 2(\sin x/2)\cos x/2}{2\cos^2 x/2 + 2(\sin x/2)\cos x/2}$

$= \lim\limits_{x \to \pi} \tan\left(\dfrac{\pi}{4} - \dfrac{\pi}{2}\right) = -1$

Since, $f(x)$ is continuous at $x = \pi$

$\therefore f(\pi) = \lim\limits_{x \to \pi} f(x) = -1$

56 (d)

$f'(1^-) = \lim\limits_{h \to 0} \dfrac{f(1 - h) - f(1)}{-h}$

$= \lim\limits_{h \to 0} \dfrac{(1 - h - 1) \cdot \sin\left(\frac{1}{1 - h - 1}\right) - 0}{-h}$

$= -\lim\limits_{h \to 0} \sin\dfrac{1}{h}$

And $f'(1^+) = \lim\limits_{h \to 0} \dfrac{f(1 + h) - f(1)}{h}$

$= \lim\limits_{h \to 0} \dfrac{(1 + h - 1)\sin\left(\frac{1}{1 + h - 1}\right) - 0}{h}$

$= \lim\limits_{h \to 0} \sin\dfrac{1}{h}$

$\therefore f'(1^-) \neq f'(1^+)$

f is not differentiable at $x = 1$

Again, now

$f'(0^+) = \lim\limits_{h \to 0} \dfrac{(0 + h - 1)\sin\left(\frac{1}{0 + h - 1}\right) - \sin 1}{h}$

$= \lim\limits_{h \to 0} \dfrac{\left[-\left\{(h - 1)\cos\left(\frac{1}{h-1}\right) \times \left(\frac{1}{(h-1)^2}\right)\right\} + \sin\left(\frac{1}{h-1}\right)\right]}{1}$

[using L 'Hospital's rule]

$= \cos 1 - \sin 1$

And $f'(0^-) = \lim\limits_{h \to 0} \dfrac{(0 - h - 1)\sin\left(\frac{1}{0 - h - 1}\right) - \sin 1}{-h}$

$= \lim\limits_{h \to 0} \dfrac{(-h - 1)\cos\left(\frac{1}{-h-1}\right)\left(\frac{1}{(-h-1)^2}\right) - \sin\left(\frac{1}{-h-1}\right)}{-1}$

[using L 'Hospital's rule]

$= \cos 1 - \sin 1$

$\Rightarrow f'(0^-) = f'(0^+)$

$\therefore f$ is differentiable at $x = 0$

57 (c)

As $f(x)$ is continuous at $x = \dfrac{\pi}{2}$

$\therefore \lim\limits_{x \to \frac{\pi}{2}^-} f(x) = \lim\limits_{x \to \frac{\pi}{2}^+} f(x)$

$\Rightarrow m\dfrac{\pi}{2} + 1 = \sin\dfrac{\pi}{2} + n \Rightarrow m\dfrac{\pi}{2} + 1 = 1 + n \Rightarrow n$

$= \dfrac{m\pi}{2}$

58 (d)

Since, $\dfrac{f(6) - f(1)}{6 - 1} \geq 2$ $\quad \left[\because f'(x) = \dfrac{y_2 - y_1}{x_2 - x_1}\right]$

$\Rightarrow f(6) - f(1) \geq 10$

$\Rightarrow f(6) + 2 \geq 10$

$\Rightarrow f(6) \geq 8$

59 (b)

We have,

$\lim\limits_{x \to a^-} f(x)\, g(x) = \lim\limits_{x \to a^-} f(x) \cdot \lim\limits_{x \to a^-} g(x) = m \times l$

$= ml$

and,

$\lim\limits_{x \to a^+} f(x)\, g(x) = \lim\limits_{x \to a^+} f(x) \lim\limits_{x \to a^+} g(x) = lm$

$\therefore \lim\limits_{x \to a^-} f(x)\, g(x) = \lim\limits_{x \to a^+} f(x)\, g(x) = lm$

Hence, $\lim\limits_{x \to a} f(x)\, g(x)$ exists and is equal to lm

60 (c)

We have,

$f'(x) = \lim\limits_{h \to 0} \dfrac{f(x + h) - f(x)}{h}$

$\Rightarrow f'(x) = \lim\limits_{h \to 0} \dfrac{f(x)f(h) - f(x)}{h}$

$\Rightarrow f'(x) = f(x)\lim\limits_{h \to 0} \dfrac{f(h) - 1}{h}$ $\quad [\because f(x + y)$

$= f(x)f(y)]$

$\Rightarrow f'(x) = f(x)\left\{\lim\limits_{h \to 0} \dfrac{1 + h\, g(h) - 1}{h}\right\}$ $\quad [\because f(x)$

$= 1 + x\, g(x)]$

$$\Rightarrow f'(x) = f(x) \lim_{h \to 0} g(h) = f(x) \cdot 1 = f(x)$$

61 (a)

We have, $f(x) = \begin{cases} x^2, & x \geq 0 \\ -x^2, & x < 0 \end{cases}$

Clearly, $f(x)$ is differentiable for all $x > 0$ and for all $x < 0$. So, we check the differentiable at $x = 0$

Now, (RHD at $x = 0$)

$$\left(\frac{d}{dx}(x)^2\right)_{x=0} = (2x)_{x=0} = 0$$

And (LHD at $= 0$)
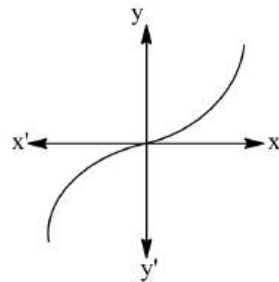
$$\left(\frac{d}{dx}(-x)^2\right)_{x=0} = (-2x)_{x=0} = 0$$

$\therefore$ (LHD at $x = 0$)=(RHD at $x = 0$)

So, $f(x)$ is differentiable for all x ie, the set of all points where $f(x)$ is differentiable is $(-\infty, \infty)$

Alternate

It is clear from the graph $f(x)$ is differentiable everywhere.

62 (a)

Since, $f'(0) = \lim_{x \to 0} \frac{f(x) - f(0)}{x - 0} = 10$

$$\Rightarrow \lim_{h \to 0} \frac{f(0 + h) - f(0)}{h} = 10$$

$$\Rightarrow f(0)\left(\lim_{h \to 0} \frac{f(h) - 1}{h}\right) = 10 \quad ...(i)$$

$[\because \quad f(0 + h) = f(0)f(h), \text{given}]$

Now, $f(0) = f(0)f(0)$

$\Rightarrow f(0) = 1$

$\therefore$ From Eq. (i)

$$\lim_{h \to 0} \frac{f(h) - 1}{h} = 10 \quad ...(ii)$$

Now, $f'(6) = \lim_{h \to 0} \frac{f(6+h) - f(6)}{h}$

$$= \lim_{x \to 0} \left(\frac{f(h) - 1}{h}\right) f(6) \quad \text{[from Eq. (ii)]}$$

$$= 10 \times 3 = 30$$

63 (a)

We have,

$$f'(a^+) = \lim_{x \to a^+} \frac{f(x) - f(0)}{x - a}$$

$$\Rightarrow f'(a^+) = \lim_{x \to a^+} \frac{|x - a|\phi(x)}{x - a}$$

$$\Rightarrow f'(a^+) = \lim_{x \to a} \frac{(x - a)}{(x - a)} \phi(x) \quad [\because x > a \therefore |x - a|$$

$$= x - a]$$

$$\Rightarrow f'(a^+) = \lim_{x \to a} \phi(x)$$

$$\Rightarrow f'(a^+) = \phi(a) \quad [\because \phi(x) \text{ is continuous at } x = a]$$

and,

$$f'(a^-) = \lim_{x \to a^-} \frac{f(x) - f(0)}{x - a}$$

$$\Rightarrow f'(a^-) = \lim_{x \to a^-} \frac{|x - a|\phi(x)}{x - a}$$

$$\Rightarrow f'(a^-) = \lim_{x \to a} \frac{(x - a)\phi(x)}{(x - a)} \quad [\because x < a \therefore |x - a|$$

$$= -(x - a)]$$

$$\Rightarrow f'(a^-) = -\lim_{x \to a} \phi(x)$$

$$\Rightarrow f'(a^-) = -\phi(a) \quad [\because$$

$\phi(x)$ is continuous at $x = a]$

64 (b)

LHL$= \lim_{h \to 0}(0 - h)e^{-\left(\frac{1}{|-h|} + \frac{1}{(-h)}\right)} = \lim_{h \to 0}(-h) = 0$

RHL$= \lim_{h \to 0}(0 + h)e^{-\left(\frac{1}{|h|} + \frac{1}{(h)}\right)} = \lim_{h \to 0} \frac{h}{e^{2/h}} = 0$

LHL$=$RHL$= f(0)$

Therefore, $f(x)$ is continuous for all x

Differentiability at $x = 0$

$$Lf'(0) = \lim_{h \to 0} \frac{(-h)e^{-\left(\frac{1}{h} - \frac{1}{h}\right)}}{(-h) - 0} = 1$$

$$Rf'(0) = \lim_{h \to 0} \frac{he^{-\left(\frac{1}{h} + \frac{1}{h}\right) - 0}}{h - 0}$$

$$= \lim_{h \to 0} \frac{1}{e^{2/h}} = 0$$

$$\Rightarrow Rf'(0)Lf'(0)$$

Therefore, $f(x)$ is not differentiable at $x = 0$

65 (d)

We have,

$$f(x) = \begin{cases} 3, & x < 0 \\ 2x + 1, & x \geq 0 \end{cases}$$

Clearly, f is continuous but not differentiable at $x = 0$

Now,

$f(|x|) = 2|x| + 1$ for all x

Clearly, $f(|x|)$ is everywhere continuous but not differentiable at $x = 0$

67 (c)

We have,

$f(x) = |x - 0.5| + |x - 1| + \tan x, 0 < x < 2$

$$\Rightarrow f(x) = \begin{cases} -2x + 1.5 + \tan x, & 0 < x < 0.5 \\ 0.5 + \tan x, & 0.5 \leq x < 1 \\ 2x - 1.5 + \tan x, & 1 \leq x < 2 \end{cases}$$

It is evident from the above definition that

$Lf'(0.5) \neq Rf'(0.5)$ and $Lf'(1) \neq Rf'(1)$

Also, the function is not continuous at $= \pi/2$. So, it cannot be differentiable thereat

68 (d)

Given, $f(x) = \begin{cases} \log_{(1-3x)}(1+3x), & \text{for } x \neq 0 \\ k, & \text{for } x = 0 \end{cases}$

$\lim_{x \to 0} f(x) = \lim_{x \to 0} \dfrac{\log(1+3x)}{\log(1-3x)}$

$= -\lim_{x \to 0} \dfrac{\log(1+3x)}{3x} \cdot \dfrac{(-3x)}{\log(1-3x)}$

$= -1$

And $f(0) = k$

$\because f(x)$ is continuous at $x = 0$

$\therefore \quad k = -1$

69 **(d)**

Since $f(x)$ is differentiable at $x = c$. Therefore, it is continuous at $x = c$

Hence, $\lim_{x \to c} f(x) = f(c)$
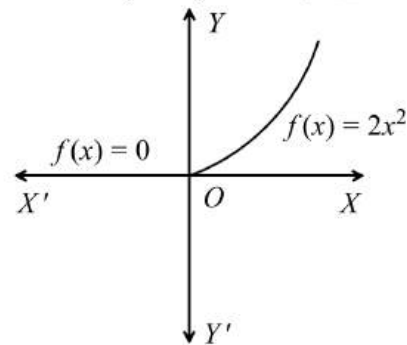
70 **(a)**

Given, $f(x) = ae^{|x|} + b\,|x|^2$

We know $e^{|x|}$ is not differentiable at $x = 0$ and $|x|^2$ is differentiable at $x = 0$

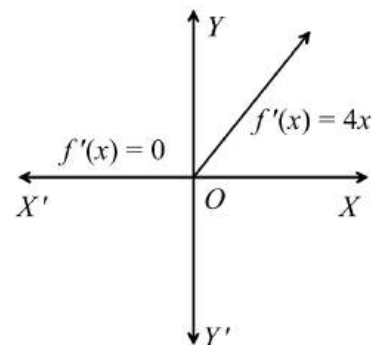$\therefore f(x)$ is differentiable at $x = 0$, if $a = 0$ and $b \in R$

71 **(a)**

We have,

$f(x) = \begin{cases} (x - x)(-x) = 0, & x < 0 \\ (x + x)x = 2x^2, & x \geq 0 \end{cases}$

(i)

(ii)

As is evident from the graph of $f(x)$ that it is continuous and differentiable for all x

Also, we have

$f''(x) = \begin{cases} 0, & x < 0 \\ 4x, & x \geq 0 \end{cases}$

Clearly, $f''(x)$ is continuous for all x but it is not differentiable at $x = 0$

72 **(b)**

Given, $f(x) = \begin{cases} \dfrac{x-1}{2x^2-7x+5}, & x \neq 1 \\ -\dfrac{1}{3}, & x = 1 \end{cases}$

$f(x) = \begin{cases} \dfrac{1}{2x-5}, & x \neq 1 \\ -\dfrac{1}{3}, & x = 1 \end{cases}$

$f'(1) = \lim_{h \to 0} \dfrac{f(1+h) - f(1)}{h}$

$= \lim_{h \to 0} \dfrac{\dfrac{1}{2(1+h)-5} - \left(-\dfrac{1}{3}\right)}{h}$

$= \lim_{h \to 0} \dfrac{\dfrac{1}{2h-3} + \dfrac{1}{3}}{h} = \lim_{h \to 0} \dfrac{3 + 2h - 3}{3h(2h-3)} = -\dfrac{2}{9}$

$Lf'(1) = \lim_{h \to 0} \dfrac{f(1-h) - f(1)}{-h}$

$= \lim_{h \to 0} \dfrac{\dfrac{1}{2(1-h)-5} - \left(-\dfrac{1}{3}\right)}{-h}$

$= \lim_{h \to 0} -\dfrac{2}{3(2h+3)} = -\dfrac{2}{9}$

$\therefore f'(1) = -\dfrac{2}{9}$

73 **(b)**

$f'(1) = \lim_{h \to 0} \dfrac{f(1+h) - f(1)}{h}$

$= \lim_{h \to 0} \dfrac{f(1+h)}{h} - \lim_{h \to 0} \dfrac{f(1)}{h}$

Given, $\lim_{h \to 0} \dfrac{f(1+h)}{h} = 5$

So, $\lim_{h \to 0} \dfrac{f(1)}{h}$ must be finite as $f'(1)$ exist and $\lim_{h \to 0} \dfrac{f(1)}{h}$ can be finite only, if $f(1) = 0$ and $\lim_{h \to 0} \dfrac{f(1)}{h} = 0$

So, $f'(1) = \lim_{h \to 0} \dfrac{f(1+h)}{h} = 5$

74 **(c)**

Since, $f(x)$ is continuous for every value of R except $\{-1, -2\}$. Now, we have to check that points

At $x = -2$

LHL$= \lim_{h \to 0} \dfrac{(-2-h)+2}{(-2-h)^2+3(-2-h)+2}$

$= \lim_{h \to 0} \dfrac{-h}{h^2 + h} = -1$

RHL$= \lim_{h \to 0} \dfrac{(-2+h)+2}{(-2+h)^2+3(-2+h)+2}$

$= \lim_{h \to 0} \dfrac{h}{h^2 - h} = -1$

$\Rightarrow$ LHL=RHL=$f(-2)$

$\therefore$ It is continuous at $x = -2$

Now, check for $x = -1$

$\text{LHL} = \lim\limits_{h \to 0} \dfrac{(-1-h)+2}{(-1-h)^2 + 3(-1-h)+2}$

$= \lim\limits_{h \to 0} \dfrac{1-h}{h^2 - h} = \infty$

$\text{RHL} = \lim\limits_{h \to 0} \dfrac{(-1+h)+2}{(-1+h)^2 + 3(-1+h)+2}$

$= \lim\limits_{h \to 0} \dfrac{1+h}{h^2 + h} = \infty$

$\Rightarrow \text{LHL} = \text{RHL} \neq f(-1)$

$\therefore$ It is not continuous at $x = -1$

The required function is continuous in $R - \{-1\}$

75 (d)

$f(0) = \lim\limits_{x \to 0} \dfrac{(e^x - 1)^2}{\sin\left(\dfrac{x}{a}\right) \log\left(1 + \dfrac{x}{4}\right)}$

$\Rightarrow \quad \lim\limits_{x \to 0} \left(\dfrac{e^x - 1}{x}\right)^2 \cdot \dfrac{\frac{x}{a} \cdot a}{\sin\frac{x}{a}} \cdot \dfrac{\frac{x}{4} \cdot 4}{\log\left(1 + \frac{x}{4}\right)} = 12$

$\Rightarrow \quad 1^2 \cdot a \cdot 4 = 12$

$\Rightarrow \quad a = 3$

76 (b)

We have,

$f(x) = \dfrac{x}{1+x} + \dfrac{x}{(x+1)(2x+1)}$

$\qquad\qquad + \dfrac{x}{(2x+1)(3x+1)} + \cdots \infty$

$\Rightarrow f(x) = \lim\limits_{n \to \infty} \sum_{r=1}^{n} \dfrac{x}{((r-1)x+1)\,(rx+1)}, \text{for } x$

$\qquad\qquad \neq 0$

$\Rightarrow f(x) = \lim\limits_{n \to \infty} \sum_{r=1}^{n} \left\{\dfrac{1}{(r-1)x+1} - \dfrac{1}{rx+1}\right\}, \text{for } x$

$\qquad\qquad \neq 0$

$\Rightarrow f(x) = \lim\limits_{n \to \infty} \left\{1 - \dfrac{2}{nx+1}\right\} = 1, \text{for } x \neq 0$

For $x = 0$, we have $f(x) = 0$

Thus, we have $f(x) = \begin{cases} 1, & x \neq 0 \\ 0, & x = 0 \end{cases}$

Clearly, $\lim\limits_{x \to 0^-} f(x) = \lim\limits_{x \to 0^+} f(x) \neq f(0)$

So, $f(x)$ is not continuous at $x = 0$

77 (b)

If possible, let $f(x) + g(x)$ be continuous. Then,

$\{f(x) + g(x)\} - f(x)$ must be continuous

$\Rightarrow g(x)$ must be continuous

This is a contradiction to the given fact that $g(x)$ is discontinuous

Hence, $f(x) + g(x)$ must be discontinuous

78 (c)

We have,

$f(x + y) = f(x)f(y)$ for all $x, y \in R$

$\therefore f(0) = f(0)f(0)$

$\Rightarrow f(0)\{f(0) - 1\} = 0$

$\Rightarrow f(0) = 1 \qquad [\because f(0) \neq 1]$

Now,

$f'(0) = 0$

$\Rightarrow \lim\limits_{h \to 0} \dfrac{f(0 + h) - f(0)}{h} = 2$

$\Rightarrow \lim\limits_{h \to 0} \dfrac{f(h) - 1}{h} = 2 \quad [\because f(0) = 1] \quad \text{....(i)}$

$\therefore f'(x) = \lim\limits_{h \to 0} \dfrac{f(x+h) - f(x)}{h}$

$\Rightarrow f'(x) = \lim\limits_{h \to 0} \dfrac{f(x)f(h) - f(x)}{h} \qquad [\because f(x+y)$

$\qquad\qquad\qquad = f(x)f(y)]$

$\Rightarrow f'(x) = f(x)\left\{\lim\limits_{h \to 0} \dfrac{f(h) - 1}{h}\right\} = 2f(x) \quad \text{[Using (i)]}$

79 (b)

We have,

$f(x) = \begin{cases} \dfrac{x^2}{|x|}, & x \neq 0 \\ 0, & x = 0 \end{cases}$

$\Rightarrow f(x) = \begin{cases} \dfrac{x^2}{2} = x, & x > 0 \\ 0, & x = 0 \\ \dfrac{x^2}{-x} = -x, & x < 0 \end{cases}$

$\Rightarrow \lim\limits_{x \to 0^-} f(x) = \lim\limits_{x \to 0} -x = 0, \ \lim\limits_{x \to 0^+} f(x) = \lim\limits_{x \to 0} x = 0$

and $f(0) = 0$

So, $f(x)$ is continuous at $x = 0$. Also, $f(x)$ is continuous for all other values of x

Hence, $f(x)$ is everywhere continuous

Clearly, $Lf'(0) = -1$ and $Rf'(0) = 1$

Therefore, $f(x)$ is not differentiable at $x = 0$

80 (b)

Since $f(x)$ is continuous at $x = 0$

$\therefore \lim\limits_{x \to 0} f(x) = f(0) \Rightarrow f(0) = 2 \quad \text{...(i)}$

Now, using L' Hospital's rule, we have

$\lim\limits_{x \to 0} \dfrac{\int_0^x f(u)\ du}{x} = \lim\limits_{x \to 0} \dfrac{f(x)}{1}$

$\qquad\qquad = f(0) \quad [\because f(x) \text{ is continuous at } x = 0]$

$\Rightarrow \lim\limits_{x \to 0} \dfrac{\int_0^x f(u)\ du}{x} = 2 \quad \text{[Using (i)]}$

82 (c)

$f'(2^+) = \lim\limits_{x \to 2^+} \left(\dfrac{f(x) - f(2)}{x - 2}\right)$

$= \lim\limits_{x \to 2^+} \dfrac{3x + 4 - (6 + 4)}{x - 2} = \lim\limits_{x \to 2} \dfrac{3x - 6}{x - 2} = 3$

83 (a)

Here, $f(x) = \begin{cases} \sin x, & x > 0 \\ 0, & x = 0 \\ -\sin x, & x < 0 \end{cases}$

RHD $= \lim_{h \to 0} \dfrac{\sin|0+h|-\sin(0)}{h}$
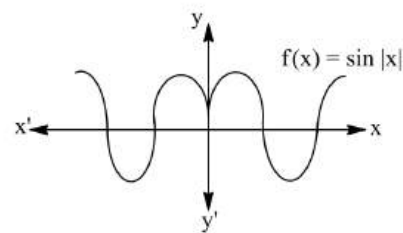
$= \lim_{h \to 0} \dfrac{\sin h}{h} = 1$

LHD $= \lim_{h \to 0} \dfrac{\sin|(0-h)|-\sin(0)}{-h}$

$= \dfrac{-\sin h}{h} = -1$

$\therefore$ LHD$\neq$RHD at $x = 0$

$\therefore$ $f(x)$ is not derivable at $x = 0$

Alternate

f(x) = sin |x|

It is clear from the graph that $f(x)$ is not differentiable at $x = 0$

84 **(b)**

We have,

$f(x) = \displaystyle\sum_{n=0}^{\infty} \dfrac{x^n}{n!} (\log_e a)^n$

$\Rightarrow f(x) = \displaystyle\sum_{n=0}^{\infty} \dfrac{(x \log_e a)^n}{n!} = \displaystyle\sum_{n=0}^{\infty} \dfrac{(\log_e a^x)^n}{n!}$

$\Rightarrow f(x) = e^{\log_e a^x} = a^x$, which is everywhere continuous and differentiable

85 **(c)**

$f(x) = [x] \cos \left[\dfrac{2x-1}{2}\right] \pi$

Since, $[x]$ is always discontinuous at all integer value, hence $f(x)$ is discontinuous for all integer value

86 **(c)**

The function f is clearly continuous for $|x| > 1$
We observe that

$\lim_{x \to -1^+} f(x) = 1, \lim_{x \to -1^-} f(x) = \dfrac{1}{4}$

Also, $\lim_{x \to \frac{1}{n}^+} f(x) = \dfrac{1}{n^2}$ and, $\lim_{x \to \frac{1}{n}^-} f(x) = \dfrac{1}{(n+1)^2}$

Thus, f is discontinuous for $x = \pm\dfrac{1}{n}, n = 1, 2, 3, \dots$

87 **(c)**

Since, $|f(x) - f(y)| \le (x-y)^2$

$\Rightarrow \lim_{x \to y} \dfrac{|f(x) - f(y)|}{|x-y|} \le \lim_{x \to y} |x-y|$

$\Rightarrow |f'(y)| \le 0$

$\Rightarrow f'(y) = 0$

$\Rightarrow f(y) =$ constant

$\Rightarrow f(y) = 0 \Rightarrow f(1) = 0$ [$\because$ $f(0) = 0$, given]

88 **(b)**

Since $\phi(x) = 2x^3 - 5$ is an increasing function on $(1, 2)$ such that $\phi(1) = -3$ and $\phi(2) = 11$
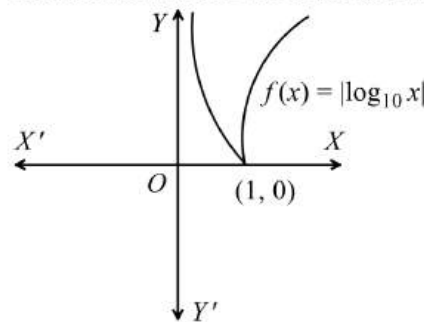Clearly, between -3 and 11 there are thirteen points where $f(x) = [2x^3 - 5]$ is discontinuous

89 **(c)**

Clearly, $[x^2 + 1]$ is discontinuous at $x = \sqrt{2}, \sqrt{3}, \sqrt{4}, \sqrt{5}, \sqrt{6}, \sqrt{7}, \sqrt{8}$
Note that it is right continuous at $x = 1$ but not left continuous at $x = 3$

90 **(a)**

As is evident from the graph of $f(x)$ that it is continuous but not differentiable at $x = 1$

$f(x) = |\log_{10} x|$

Now,

$f''(1^+) = \lim_{x \to 1^+} \dfrac{f(x) - f(1)}{x - 1}$

$\Rightarrow f''(1^+) = \lim_{h \to 0} \dfrac{f(1+h) - f(1)}{h}$

$\Rightarrow f''(1^+) = \lim_{h \to 0} \dfrac{\log_{10}(1+h) - 0}{h}$

$\Rightarrow f''(1^+) = \lim_{h \to 0} \dfrac{\log(1+h)}{h. \log_e 10} = \dfrac{1}{\log_e 10} = \log_{10} e$

$f''(1^-) = \lim_{x \to 1^+} \dfrac{f(x) - f(1)}{x - 1}$

$\Rightarrow f''(1^-) = \lim_{h \to 0} \dfrac{f(1-h) - f(1)}{h}$

$\Rightarrow f''(1^-) = \lim_{h \to 0} \dfrac{\log_{10}(1-h)}{h} = \lim_{h \to 0} \dfrac{\log_e(1-h)}{h \log_e 10}$

$= -\log_{10} e$

91 **(b)**

It can be easily seen from the graph of $f(x) = |\cos x|$ that it is everywhere continuous but not differentiable at odd multiples of $\pi/2$

92 **(d)**

We have,

$\lim_{x \to 4^-} f(x) = \lim_{h \to 0} f(4-h) = \lim_{h \to 0} \dfrac{4 - h - 4}{|4 - h - 4|} + a$

$\Rightarrow \lim_{x \to 4^-} f(x) = \lim_{h \to 0} -\dfrac{h}{h} + a = a - 1$

$$\Rightarrow \lim_{x\to 4^-} f(x) = \lim_{h\to 0} f(4+h) = \lim_{h\to 0} \frac{4+h-4}{|4+h-4|} + b$$
$$= b+1$$

and, $f(4) = a+b$

Since $f(x)$ is continuous at $x = 4$. Therefore,

$$\lim_{x\to 4^-} f(x) = f(4) = \lim_{x\to 4^+} f(x)$$

$$\Rightarrow a-1 = a+b = b+1 \Rightarrow b = -1 \text{ and } a = 1$$

93 (b)

We have,

$$f(x) = \begin{cases} \dfrac{2^x - 1}{\sqrt{1+x} - 1}, & -1 \le x < \infty, \quad x \ne 0 \\ k, & x = 0 \end{cases}$$

Since, $f(x)$ is continuous everywhere

$$\therefore \lim_{x\to 0^-} f(x) = f(0) \quad \text{...(i)}$$

Now, $\lim_{x\to 0^-} f(x) = \lim_{h\to 0} \dfrac{2^{(0-h)}-1}{\sqrt{1+(0-h)}-1}$

$$= \lim_{h\to 0} \frac{2^{-h}-1}{\sqrt{1-h}-1}$$

$$= \lim_{h\to 0} \frac{-2^{-h}\log_e 2}{\frac{-1}{2\sqrt{1-h}}} \quad \text{[by L' Hospital's rule]}$$

$$= 2 \lim_{h\to 0} 2^{-h} \log_e 2 \sqrt{1-h}$$

$$= 2 \log_e 2$$

From Eq. (i),

$$f(0) = 2\log_e 2 = \log_e 4$$

95 (b)

We have,

$$\lim_{x\to 0^-} f(x) = \lim_{h\to 0} f(-h) = \lim_{h\to 0} \frac{e^{-1/h}-1}{e^{-1/h}+1} = -1$$

and,

$$\lim_{x\to 0^+} f(x) = \lim_{h\to 0} f(h) = \lim_{x\to 0} \frac{e^{1/h}-1}{e^{1/h}+1} = \lim_{h\to 0} \frac{e^{-1/h}}{e^{-1/h}}$$

$$= 1$$

$$\therefore \lim_{x\to 0^-} f(x) \ne \lim_{x\to 0^+} f(x)$$

Hence, $f(x)$ is not continuous at $x = 0$

96 (c)

LHL$= \lim_{x\to 2^-} f(x) = \lim_{h\to 0} 1 + (2-h) = 3$

RHL$= \lim_{x\to 2^+} f(x) = \lim_{h\to 0} 5 - (2+h) = 3, \quad f(2) = 3$

Hence, f is continuous at $x = 2$

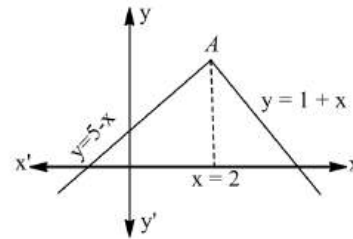Now, $Rf''(2) = \lim_{h\to 0} \dfrac{f(2+h)-f(2)}{h}$

$$= \lim_{h\to 0} \frac{5-(2+h)-3}{h} = -1$$

$Lf''(2) = \lim_{h\to 0} \dfrac{f(2-h)-f(2)}{-h}$

$$= \lim_{h\to 0} \frac{1+(2-h)-3}{-h} = 1$$

$\therefore Rf''(2) \ne Lf''(2)$

$\therefore f$ is not differentiable at $x = 2$

Alternate

It is clear from the graph that $f(x)$ is continuous everywhere also it is differentiable everywhere except at $x = 2$

97 (d)

We have,

$f(x+y) = f(x)f(y)$ for all $x, y \in R$

Putting $x = 1, y = 0$, we get

$f(0) = f(1)f(0) \Rightarrow f(0)(1-f(1)) = 0$

$$\Rightarrow f(1) = 1 \qquad [\because f(0) \ne 0]$$

Now,

$f'(1) = 2$

$$\Rightarrow \lim_{h\to 0} \frac{f(1+h)-f(1)}{h} = 2$$

$$\Rightarrow \lim_{h\to 0} \frac{f(1)f(h)-f(1)}{h} = 2$$

$$\Rightarrow f(1) \lim_{h\to 0} \frac{f(h)-1}{h} = 2$$

$$\Rightarrow \lim_{h\to 0} \frac{f(h)-1}{h} = 2 \quad [\text{Using } f(1)=1] \quad \text{...(i)}$$

$$\therefore f'(4) = \lim_{h\to 0} \frac{f(4+h)-f(4)}{h}$$

$$\Rightarrow f'(4) = \lim_{h\to 0} \frac{f(4)f(h)-f(4)}{h}$$

$$\Rightarrow f'(4) = \left\{ \lim_{h\to 0} \frac{f(h)-1}{h} \right\} f(4)$$

$$\Rightarrow f'(4) = 2 f(4) \qquad [\text{From (i)}]$$

$$\Rightarrow f'(4) = 2 \times 4 = 8$$

98 (d)

We have,

$$\lim_{x\to 1^-} g(x) = \lim_{x\to 1^+} g(x) = 1 \text{ and } g(1) = 0$$

So, $g(x)$ is not continuous at $x = 1$ but $\lim_{x\to 1} g(x)$ exists

We have,

$$\lim_{x\to 1^-} f(x) = \lim_{h\to 0} f(1-h) = \lim_{h\to 0} [1-h] = 0$$

and,

$$\lim_{x\to 1^+} f(x) = \lim_{h\to 0} f(1+h) = \lim_{h\to 0} [1+h] = 1$$

So, $\lim_{x\to 1} f(x)$ does not exist and so $f(x)$ is not continuous at $x = 1$

We have, $gof(x) = g(f(x)) = g([x]) = 0$, for all $x \in R$

So, gof is continuous for all x

We have,

$fog(x) = f(g(x))$

$$\Rightarrow fog(x) = \begin{cases} f(0), & x \in Z \\ f(x^2), & x \in R - Z \end{cases}$$

$$\Rightarrow fog(x) = \begin{cases} 0, & x \in Z \\ [x^2], & x \in R - Z \end{cases}$$

Which is clearly not continuous

99 (d)

At $x = 1$,

RHD $= \lim_{h \to 0^+} \dfrac{f(1+h)-f(1)}{h}$

$= \lim_{h \to 0} \dfrac{2-(1+h)-(2-1)}{h} = -1$

LHD $= \lim_{h \to 0^-} \dfrac{f(1-h)-f(1)}{-h}$

$= \lim_{h \to 0} \dfrac{(1-h)-(2-1)}{-h} = 1$

$\therefore$ LHD$\neq$RHD

100 (d)

Given, $f(x) = |x| + \dfrac{|x|}{x}$

Let $f_1(x) = |x|$, $f_2(x) = \dfrac{|x|}{x}$

1.   LHL $= \lim_{x \to 0^-} f_1(x) = \lim_{x \to 0^-} |x| = 0$

And RHL $\lim_{x \to 0^+} f_1(x) = \lim_{x \to 0^+} |x| = 0$

Here, LHL=RHL=$f(0)$, $f_1(x)$ is continuous

2.   LHL $= \lim_{x \to 0^-} \dfrac{|x|}{x} = \lim_{h \to 0} \dfrac{|0-h|}{0-h} = -1$

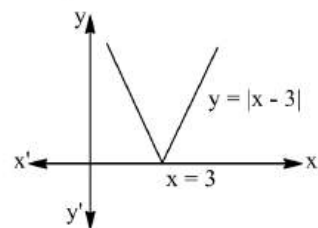RHL $= \lim_{x \to 0^+} \dfrac{|x|}{x} = \lim_{h \to 0} \dfrac{|0+h|}{h} = 1$

$\therefore$ LHL$\neq$RHL, $f_2(x)$ is discontinuous

Hence, $f(x)$ is discontinuous at $x = 0$

101 (a)

From the graph it is clear that $f(x)$ is continuous everywhere but not differentiable at $x = 3$

102 (b)

Given, $f(x) = \begin{cases} \dfrac{2x-3}{2x-3}, & \text{if } x > \dfrac{3}{2} \\ \dfrac{-(2x-3)}{2x-3}, & \text{if } x < \dfrac{3}{2} \end{cases}$

$= \begin{cases} 1, & \text{if } x > \dfrac{3}{2} \\ -1, & \text{if } x < \dfrac{3}{2} \end{cases}$

Now, RHL $= \lim_{x \to \frac{3}{2}^+} f(x) = \lim_{x \to \frac{3}{2}^+} 1 = 1$

And LHL $= \lim_{x \to \frac{3}{2}^-} f(x) = \lim_{x \to \frac{3}{2}^-} (-1) = -1$

$\because$ RHL$\neq$LHL

$\therefore$ $f(x)$ is discontinuous at $x = \dfrac{3}{2}$

103 (c)

Since the functions $(\log t)^2$ and $\dfrac{\sin t}{t}$ are not defined on $(-1, 2)$. Therefore, the functions in options (a) and (b) are not defined on $(-1, 2)$

The function $g(t) = \dfrac{1-t+t^2}{1+t+t^2}$ is continuous on $(-1, 2)$ and

$f(x) = \int_0^x \dfrac{1-t+t^2}{1+t+t^2} dt$ is the integral function of $g(t)$

Therefore, $f(x)$ is differentiable on $(-1, 2)$ such that $f'(x) = g(x)$

104 (c)

Since, $f(x) = \dfrac{1-\tan x}{4x-\pi}$

Now, $\lim_{x \to \pi/4} f(x) = \lim_{x \to \pi/4} \left(\dfrac{1-\tan x}{4x-\pi} \right)$

$= \lim_{x \to \pi/4} \left(\dfrac{-\sec^2 x}{4} \right) = -\dfrac{1}{2}$

Since, $f(x)$ is continuous at

$x = \dfrac{\pi}{4}$

$\therefore$ $\lim_{x \to \pi/4} f(x) = f\left(\dfrac{\pi}{4}\right) = -\dfrac{1}{2}$

105 (a)

$\lim_{x \to 0} f(x) = \lim_{x \to 0} \dfrac{1-\cos x}{x} = \lim_{x \to 0} \dfrac{2\sin^2 \frac{x}{2}}{4\left(\frac{x}{2}\right)^2} \cdot x = 0$

Also, $f(0) = k$

For, $\lim_{x \to 0} f(x) = f(0) \Rightarrow k = 0$

106 (a)

We have,

$f(x) = |x| + |x-1|$

$\Rightarrow f(x) = \begin{cases} -2x+1, & x < 0 \\ x-x+1, & 0 \le x < 1 \\ x+x-1, & x \ge 1 \end{cases}$

$\Rightarrow f(x) = \begin{cases} -2x+1, & x < 0 \\ 1, & 0 \le x < 1 \\ 2x-1, & x \ge 1 \end{cases}$

Clearly, $\lim_{x \to 0^-} f(x) = 1 = \lim_{x \to 0^+} f(x)$ and

$\lim_{x \to 1^-} f(x) = \lim_{x \to 1^+} f(x)$

So, $f(x)$ is continuous at $x = 0, 1$

107 (d)

$f(0) = \lim_{x \to 0} \dfrac{2x - \sin^{-1} x}{2x + \tan^{-1} x}$

$$= \lim_{x \to 0} \frac{2 - \frac{\sin^{-1} x}{x}}{2 + \frac{\tan^{-1} x}{x}}$$

$$= \frac{2 - 1}{2 + 1} = \frac{1}{3}$$

109 (b)

$$f'(1) = \lim_{h \to 0} \frac{f(1+h) - f(1)}{h}$$

$$= \lim_{h \to 0} \frac{\frac{1+h-1}{2(1+h)^2 - 7(1+h) + 5} - \left(\frac{1}{3}\right)}{h}$$

$$= \lim_{h \to 0} \frac{\left(\frac{1}{2h-3} + \frac{1}{3}\right)}{h} = \lim_{h \to 0} \left(\frac{2h}{3h(2h-3)}\right) = -\frac{2}{9}$$

110 (a)

LHL $= \lim_{h \to 0} f\left(-\frac{\pi}{2} - h\right) = \lim_{h \to 0} 2 \cos\left(-\frac{\pi}{2} - h\right) = 0$

RHL $= \lim_{h \to 0} f\left(-\frac{\pi}{2} + h\right) = \lim_{h \to 0} 2 a \sin\left(-\frac{\pi}{2} + h\right) + b$

$= -a + b$

Since, function is continuous.

$\therefore$ RHL=LHL $\Rightarrow a = b$

From the given options only (a) ie, $\left(\frac{1}{2}, \frac{1}{2}\right)$ satisfies

this condition

111 (a)

We have,

$f'(0) = 3$

$\Rightarrow \lim_{x \to 0} \frac{f(x) - f(0)}{x - 0} = 3$

$\Rightarrow \lim_{h \to 0} \frac{f(0+h) - f(0)}{h}$

$\qquad = 3$ [Using: (RHD at $x = 0$) $= 3$]

$\Rightarrow \lim_{h \to 0} \frac{f(0)f(h) - f(0)}{h}$

$\qquad = 3$ $\begin{bmatrix} \because f(x+y) = f(x)f(y) \\ \therefore f(0+h) = f(0)f(h) \end{bmatrix}$

$\Rightarrow f(0) \left(\lim_{h \to 0} \frac{f(h)-1}{h}\right) = 3$ ...(i)

Now, $f(x+y) = f(x)f(y)$ for all $x, y \in R$

$\Rightarrow f(0) = f(0)f(0)$

$\Rightarrow f(0)\{1 - f(0)\} = 0 \Rightarrow f(0) = 1$

Putting $f(0) = 1$ in (i), we get

$\lim_{h \to 0} \frac{f(h)-1}{h} = 3$ ...(ii)

Now,

$f'(5) = \lim_{h \to 0} \frac{f(5+h) - f(5)}{h}$

$\Rightarrow f'(5) = \lim_{h \to 0} \frac{f(5)f(h) - f(5)}{h}$

$\Rightarrow f'(5) = \left\{\lim_{h \to 0} \frac{f(h)-1}{h}\right\} f(5) = 3 \times 2 = 6$

[Using (ii)]

112 (c)

We have,

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$\Rightarrow f(x)' = \lim_{h \to 0} \frac{f(x) + f(h) - f(x)}{h}$$

$$\Rightarrow f(x)' = \lim_{h \to 0} \frac{f(h)}{h}$$

$$\Rightarrow f(x)' = \lim_{h \to 0} \frac{h \, g(h)}{h} \lim_{h \to 0} g(h) = g(0) \quad [$$

$$\because g \text{ is conti. at } x = 0]$$

113 (b)

The domain of $f(x)$ is $[2, \infty)$

We have,

$$f(x) = \sqrt{\frac{\left(\sqrt{2x-4}\right)^2}{2} + 2 + 2\sqrt{2x-4}}$$

$$+ \sqrt{\frac{\left(\sqrt{2x-4}\right)^2}{2} + 2 - 2\sqrt{2x-4}}$$

$$\Rightarrow f(x) = \frac{1}{\sqrt{2}} \sqrt{\left(\sqrt{2x-4}\right)^2 + 4\sqrt{2x-4} + 4}$$

$$+ \frac{1}{\sqrt{2}} \sqrt{\left(\sqrt{2x-4}\right)^2 - 4\sqrt{2x-4} + 4}$$

$$\Rightarrow f(x) = \frac{1}{\sqrt{2}} \left|\sqrt{2x-4} + 2\right| + \frac{1}{\sqrt{2}} \left|\sqrt{2x-4} - 2\right|$$

$$\Rightarrow f(x) = \begin{cases} \frac{1}{\sqrt{2}} \times 4, & \text{if } \sqrt{2x-4} < 2 \\ \sqrt{2} \cdot \sqrt{2x-4}, & \text{if } \sqrt{2x-4} \geq 2 \end{cases}$$

$$\Rightarrow f(x) = \begin{cases} 2\sqrt{2}, & \text{if } x \in [2, 4) \\ 2\sqrt{x-2}, & \text{if } x \in [4, \infty) \end{cases}$$

Hence, $f'(x) = \begin{cases} 0 & \text{if } x \in [2, 4) \\ \frac{1}{\sqrt{x-2}} & \text{if } x \in (4, \infty) \end{cases}$

114 (c)

We have,

$$\lim_{x \to 0} \frac{f(x) - f(0)}{x - 0} = \lim_{x \to 0} \frac{x^2 \sin\left(\frac{1}{x}\right)}{x} = \lim_{x \to 0} x \sin\frac{1}{x} = 0$$

So, $f(x)$ is differentiable at $x = 0$ such that

$f'(0) = 0$

For $x \neq 0$, we have

$$f'(x) = 2x \sin\left(\frac{1}{x}\right) + x^2 \cos\left(\frac{1}{x}\right)\left(-\frac{1}{x^2}\right)$$

$$\Rightarrow f'(x) = 2x \sin\frac{1}{x} - \cos\frac{1}{x}$$

$$\Rightarrow \lim_{x \to 0} f'(x) = \lim_{x \to 0} 2x \sin\frac{1}{x} - \lim_{x \to 0} \cos\left(\frac{1}{x}\right)$$

$$= 0 - \lim_{x \to 0} \cos\left(\frac{1}{x}\right)$$

Since $\lim_{x \to 0} \cos\left(\frac{1}{x}\right)$ does not exist

$\therefore \lim_{x \to 0} f'(x)$ does not exist

Hence, $f'(x)$ is not continuous at $x = 0$

115 (c)

We have,

$$f(x) = \begin{cases} \dfrac{x}{\sqrt{x^2}}, & x \neq 0 \\ 0, & x = 0 \end{cases}$$

$$\Rightarrow f(x) = \begin{cases} \dfrac{x}{|x|}, & x \neq 0 \\ 0, & x = 0 \end{cases} = \begin{cases} 1, & x > 0 \\ -1, & x < 0 \\ 0, & x = 0 \end{cases}$$

Clearly, $f(x)$ is not continuous at $x = 0$

117 (c)

Given, $\lim\limits_{x \to 0}\left[(1 + 3x)^{\frac{1}{x}}\right] = k$

$\therefore \quad e^3 = k$

118 (b)

For $x > 2$, we have

$$f(x) = \int_0^x \{5 + |1 - t|\}dt$$

$$\Rightarrow f(x) = \int_0^1 (5 + (1-t)dt + \int_1^x (5 - (1-t)\,dt$$

$$\Rightarrow f(x) = \int_0^1 (6 - t)dt + \int_1^x (4 + t)dt$$

$$\Rightarrow f(x) = \left[6t - \frac{t^2}{2}\right]_0^1 + \left[4t + \frac{t^2}{2}\right]_1^x$$

$$\Rightarrow f(x) = 1 + 4x + \frac{x^2}{2}$$

Thus, we have

$$f(x) = \begin{cases} 5x + 1, & \text{if } x \leq 2 \\ \dfrac{x^2}{2} + 4x + 1, & \text{if } x > 2 \end{cases}$$

Clearly, $f(x)$ is everywhere continuous and differentiable except possibly at $x = 2$

Now,

$$\lim_{x \to 2^-} f(x) = \lim_{x \to 2} 5x + 1 = 11$$

and,

$$\lim_{x \to 2^+} f(x) = \lim_{x \to 2} \left(\frac{x^2}{2} + 4x + 1\right) = 11$$

$\therefore \quad \lim\limits_{x \to 2^-} f(x) = \lim\limits_{x \to 2^+} f(x)$

So, $f(x)$ is continuous at $x = 2$

Also, we have (LHD at $x = 2$) $= \lim\limits_{x \to 2^-} f'(x) = \lim\limits_{x \to 2} 5 = 5$

119 (b)

The given function is clearly continuous at all points except possibly at $x = \pm 1$

For $f(x)$ to be continuous at $x = 1$, we must have

$$\lim_{x \to 1^-} f(x) = \lim_{x \to 1^+} f(x) = f(1)$$

$$\Rightarrow \lim_{x \to 1} ax^2 + b = \lim_{x \to 1} \frac{1}{|x|}$$

$$\Rightarrow a + b = 1 \qquad \ldots\text{(i)}$$

Clearly, $f(x)$ is differentiable for all x, except possibly at $x = \pm 1$. As $f(x)$ is an even function, so we need to check its differentiability at $x = 1$ only
For $f(x)$ to be differentiable at $x = 1$, we must have

$$\lim_{x \to 1^-} \frac{f(x) - f(1)}{x - 1} = \lim_{x \to 1^+} \frac{f(x) - f(1)}{x - 1}$$

$$\Rightarrow \lim_{x \to 1} \frac{ax^2 + b - 1}{x - 1} = \lim_{x \to 1} \frac{\frac{1}{|x|} - 1}{x - 1}$$

$$\Rightarrow \lim_{x \to 1} \frac{ax^2 - a}{x - 1} = \lim_{x \to 1} \frac{\frac{1}{x} - 1}{x - 1} \qquad \begin{bmatrix} \because a + b = 1 \\ \therefore b - 1 = -a \end{bmatrix}$$

$$\Rightarrow \lim_{x \to 1} a(x + 1) = \lim_{x \to 1} \frac{-1}{x}$$
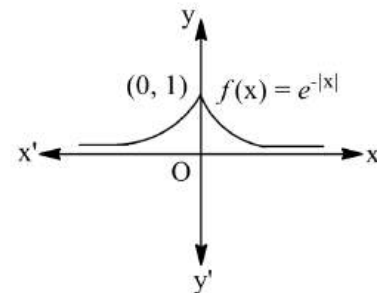
$$\Rightarrow 2a = -1 \Rightarrow a = -1/2$$

Putting $a = -1/2$ in (i), we get $b = 3/2$

120 (c)

At no point, function is continuous

121 (a)

It is clear from the figure that $f(x)$ is continuous everywhere and not differentiable at $x = 0$ due to sharp edge

122 (c)

$$f(x) = \frac{\sqrt{a^2 - ax + x^2} - \sqrt{a^2 + ax + x^2}}{\sqrt{a + x} - \sqrt{a - x}}$$

$$\times \frac{\sqrt{a^2 - ax + x^2} + \sqrt{a^2 + ax + x^2}}{\sqrt{a^2 - ax + x^2} + \sqrt{a^2 + ax + x^2}}$$

$$\times \frac{\sqrt{a + x} + \sqrt{a - x}}{\sqrt{a + x} + \sqrt{a - x}}$$

$$\Rightarrow \lim_{x \to 0} f(x)$$

$$= \lim_{x \to 0} \frac{-2ax(\sqrt{a + x} + \sqrt{a - x})}{2x\left(\sqrt{a^2 - ax + x^2} + \sqrt{a^2 + ax + x^2}\right)}$$

$$= \frac{-a(2\sqrt{a})}{(a + a)} = -\sqrt{a}$$

123 (b)

Given, $f(x) = \begin{cases} \dfrac{1 - \cos 4x}{8x^2}, & x \neq 0 \\ k & x = 0 \end{cases}$

LHL $= \lim\limits_{x \to 0^-} f(x)$

$$= \lim_{h \to 0} \frac{1 - \cos 4(0 - h)}{8(0 - h)^2}$$

$= \lim_{h \to 0} \frac{1 - \sin 4h}{8h^2}$

$= \lim_{h \to 0} \frac{4 \sin 4h}{16h} = 1$ [by L 'Hospital's rule]

Since, $f(x)$ is continuous at $x = 0$

$\therefore \quad f(0) =$ LHL $\Rightarrow \quad k = 1$

124 (d)

Given, $f(x) = |x - 1| + |x - 2| + \cos x$

Since, $|x - 1|, |x - 2|$ and $\cos x$ are continuous in $[0, 4]$

$\therefore f(x)$ being sum of continuous functions is also continuous

125 (c)

If function $f(x)$ is continuous at $x = 0$, then

$f(0) = \lim_{x \to 0} f(x)$

$\therefore \quad f(0) = k = \lim_{x \to 0} x \sin \frac{1}{x}$

$\Rightarrow k = 0 \qquad \left[\because -1 \le \sin \frac{1}{x} \le 1 \right]$

126 (b)

We have,

$h(x) = \{f(x)\}^2 + \{g(x)\}^2$

$\Rightarrow h'(x) = 2f(x)2f'(x) + 2\,g(x)\,g'(x)$

Now,

$f'(x) = g(x)$ and $f''(x) = -f(x)$

$\Rightarrow f''(x) = g'(x)$ and $f''(x) = -f(x)$

$\Rightarrow -f(x) = g'(x)$

Thus, we have

$f'(x) = g(x)$ and $g'(x) = -f(x)$

$\therefore h'(x) = -2\,g(x)g'(x) + 2\,g\,(x)g'(x) = 0$, for all x

$\Rightarrow h(x) =$ Constant for all x

But, $h(5) = 11$. Hence, $h(x) = 11$ for all x

127 (a)

$f(x) = |x|^3 = \begin{cases} 0, & x = 0 \\ x^3, & x > 0 \\ -x^3, & x < 0 \end{cases}$

Now, $Rf'(0) = \lim_{h \to 0} \frac{h^3 - 0}{h} = 0$

And $Lf'(0) = \lim_{h \to 0} \frac{-h^3 - 0}{-h} = 0$

$\therefore Rf'(0) = Lf'(0) = 0$

$\therefore f'(0) = 0$

128 (b)

We have,

(LHL at $x = 0$) $= \lim_{n \to 0^-} f(x) = \lim_{h \to 0} f(0 - h)$

$\Rightarrow$ (LHL at $x = 0$) $= \lim_{n \to 0} \sin^{-1}(\cos(-h))$

$= \lim_{h \to 0} \sin^{-1}(\cos h)$

$\Rightarrow$ (LHL at $x = 0$) $= \sin^{-1} 1 = \pi/2$

(RHL at $x = 0$) $= \lim_{x \to 0^+} f(x)$

$\Rightarrow$ (RHL at $x = 0$) $= \lim_{h \to 0} f(0 + h)$

$= \lim_{h \to 0} \sin^{-1}(\cos h)$

$\Rightarrow$ (RHL at $x = 0$) $= \sin^{-1}(1) = \pi/2$

and, $f(0) = \sin^{-1}(\cos 0) = \sin^{-1}(1) = \pi/2$

$\therefore$ (LHL at $x = 0$) $=$ (RHL at $x = 0$) $= f(0)$

So, $f(x)$ is continuous at $x = 0$

Now,

$f'(x) = \frac{-\sin x}{\sqrt{1 - \cos^2 x}} = \frac{\sin x}{|\sin x|}$

$= \begin{cases} \dfrac{-\sin x}{-\sin x} = 1, x < 0 \\ \dfrac{-\sin x}{\sin x} = -1, x > 0 \end{cases}$

$\therefore$ (LHD at $x = 0$) $= 1$ and (RHD at $x = 0$) $= -1$

Hence, $f(x)$ is not differentiable at $x = 0$

129 (d)

For any $x \ne 1, 2$, we find that $f(x)$ is the quotient of two polynomials and a polynomial is everywhere continuous. Therefore, $f(x)$ is continuous for all $x \ne 1, 2$

Continuity at $x = 1$:

We have,

$\lim_{x \to 1^-} f(x) = \lim_{h \to 0} f(1 - h)$

$\Rightarrow \lim_{x \to 1^-} f(x)$

$= \lim_{h \to 0} \frac{(1 - h - 2)(1 - h + 2)(1 - h + 1)(1 - h - }{|(1 - h - 1)(1 - h - 2)|}$

$\Rightarrow \lim_{x \to 1^-} f(x) = \lim_{h \to 0} \frac{(3 - h)(2 - h)(-1 - h)(-h)}{|(-h)(-1 - h)|}$

$\Rightarrow \lim_{x \to 1^-} f(x) = \lim_{h \to 0} \frac{(3 - h)(2 - h)h(h + 1)}{h(h + 1)} = 6$

and,

$\lim_{x \to 1^+} f(x) = \lim_{h \to 0} f(1 + h)$

$\lim_{x \to 1^+} f(x)$

$= \lim_{h \to 0} \frac{(1 + h - 2)(1 + h + 2)(1 + h + 1)(1 + h - }{|(1 + h - 1)(1 + h - 2)|}$

$\lim_{x \to 1^+} f(x) = \lim_{h \to 0} \frac{(h - 1)(3 + h)(2 + h)(h)}{|h(h - 1)|}$

$\lim_{x \to 1^+} f(x) = -\lim_{h \to 0} \frac{(h - 1)(3 + h)(2 + h)h}{h(1 - h)} = -6$

$\therefore \lim_{x \to 1^-} f(x) \ne \lim_{x \to 1^+} f(x)$

So, $f(x)$ is not continuous at $x = 1$

Similarly, $f(x)$ is not continuous at $x = 2$

130 (b)

Let $f(x) = \frac{g(x)}{h(x)} = \frac{x}{1 + |x|}$

It is clear that $g(x) = x$ and $h(x) = 1 + |x|$ are differentiable on $(-\infty, \infty)$ and $(-\infty, 0) \cup (0, \infty)$ respectively

Thus, $f(x)$ is differentiable on $(-\infty, 0) \cup (0, \infty)$. Now, we have to check the differentiable at $x = 0$

$\therefore \lim\limits_{x \to 0} \dfrac{f(x) - f(0)}{x - 0} = \lim\limits_{x \to 0} \dfrac{\frac{x}{1+|x|} - 0}{x} = \lim\limits_{x \to 0} \dfrac{1}{1 + |x|}$

$= 1$

Hence, $f(x)$ is differntaible on $(-\infty, \infty)$

131 (b)

At $x = 0$,

LHL $= \lim\limits_{h \to 0} \dfrac{1}{1 - e^{-1/(0-h)}} = \lim\limits_{h \to 0} \dfrac{1}{1 - e^{1/h}} = 0$

RHL $= \lim\limits_{h \to 0} \dfrac{1}{1 - e^{-1/(0+h)}} = \lim\limits_{h \to 0} \dfrac{1}{1 - e^{1/h}} = 1$

$\therefore$ FUnction is not continuous at $x = 0$

132 (a)

We have,

$fog = I$

$\Rightarrow fog(x) = x$ for all x

$\Rightarrow f'\big(g(x)\big)g'(x) = 1$ for all x

$\Rightarrow f'\big(g(a)\big) = \dfrac{1}{g'(a)} = \dfrac{1}{2} \Rightarrow f'(b)$

$= \dfrac{1}{2} \quad [\because f(a) = b]$

133 (a)

Since, $\lim\limits_{x \to 0} f(x) = f(0)$

$\Rightarrow \lim\limits_{x \to 0} \dfrac{\sin \pi x}{5x} = k$

$\Rightarrow (1)\dfrac{\pi}{5} = k \Rightarrow k = \dfrac{\pi}{5} \quad \left[\because \lim\limits_{x \to 0} \dfrac{\sin x}{x} = 1\right]$

134 (d)

Given, $f(x) = [x], x \in (-3.5, 100)$

As we know greatest integer is discontinuous on integer values.

In given interval, the integer values are

$(-3, -2, -1, 0, \ldots, 99)$

$\therefore$ Total numbers of integers are 103.

135 (a)

LHL $= \lim\limits_{h \to 0} f(0 - h)$

$= \lim\limits_{h \to 0} \dfrac{e^{-1/h} - 1}{e^{-1/h} + 1} = -1 \quad \left[\because \lim\limits_{h \to 0} \dfrac{1}{e^{1/h}} = 0\right]$

RHL $= \lim\limits_{h \to 0} f(0 + h) = \lim\limits_{h \to 0} \dfrac{e^{1/h} - 1}{e^{1/h} + 1}$

$= \lim\limits_{h \to 0} \dfrac{1 - \frac{1}{e^{1/h}}}{1 + \frac{1}{e^{1/h}}} = 1$

$\therefore$ LHL $\neq$ RHL

So, limit does not exist at $x = 0$

136 (d)

We have,

$f(x) = x^4 + \dfrac{x^4}{1 + x^4} + \dfrac{x^4}{(1 + x^4)} + \cdots$

$\Rightarrow f(x) = \dfrac{x^4}{1 - \frac{1}{1+x^4}} = 1 + x^4, \text{if } x \neq 0$

Clearly, $f(x) = 0$ at $x = 0$

Thus, we have

$f(x) = \begin{cases} 1 + x^4, & x \neq 0 \\ 0, & x = 0 \end{cases}$

Clearly, $\lim\limits_{x \to 0^-} f(x) = \lim\limits_{x \to 0^+} f(x) = 1 \neq f(0)$

So, $f(x)$ is neither continuous nor differentiable at $x = 0$

137 (a)

We have,

$f(x) = \begin{cases} 1 + x, & 0 \leq x \leq 2 \\ 3 - x, & 2 < x \leq 3 \end{cases}$

$\therefore g(x) = fof(x)$

$\Rightarrow f(x) = f\big(f(x)\big)$

$\Rightarrow g(x) = \begin{cases} f(1 + x), & 0 \leq x \leq 2 \\ f(3 - x), & 2 < x \leq 3 \end{cases}$

$\Rightarrow g(x) = \begin{cases} 1 + (1 + x), & 0 \leq x \leq 1 \\ 3 - (1 + x), & 1 < x \leq 2 \\ 1 + (3 - x), & 2 < x \leq 3 \end{cases}$

$\Rightarrow g(x) = \begin{cases} 2 + x, & 0 \leq x \leq 1 \\ 2 - x, & 1 < x \leq 2 \\ 4 - x, & 2 < x \leq 3 \end{cases}$

Clearly, $g(x)$ is continuous in $(0, 1) \cup (1, 2) \cup (2, 3)$ except possibly at $x = 0, 1, 2$ and 3

We observe that

$\lim\limits_{x \to 0^+} g(x) = \lim\limits_{x \to 0^+} (2 + x) = 2 = g(0)$

and $\lim\limits_{x \to 3^-} g(x) = \lim\limits_{x \to 3^-} 4 - x = 1 = g(3)$

Therefore, $g(x)$ is right continuous at $x = 0$ and left continuous at $x = 3$

At $x = 1$, we have

$\lim\limits_{x \to 1^-} g(x) = \lim\limits_{x \to 1^-} 2 + x = 3$

and, $\lim\limits_{x \to 1^+} g(x) = \lim\limits_{x \to 1^+} 2 - x = 1$

$\therefore \lim\limits_{x \to 1^+} g(x) \neq \lim\limits_{x \to 1^-} g(x)$

So, $g(x)$ is not continuous at $x = 1$

At $x = 2$, we have

$\lim\limits_{x \to 2^-} g(x) = \lim\limits_{x \to 2^-} (2 - x) = 0$

and,

$\lim\limits_{x \to 2^+} g(x) = \lim\limits_{x \to 2^+} (4 - x) = 0$

$\therefore \lim\limits_{x \to 2^-} g(x) \neq \lim\limits_{x \to 2^+} g(x)$

So, $g(x)$ is not continuous at $x = 2$

Hence, the set of points of discontinuity of $g(x)$ is $\{1, 2\}$

138 (b)

Since $g(x)$ is the inverse of function $f(x)$

$\therefore gof(x) = I(x)$, for all x

Now, $gof(x) = I(x)$, for all x

$\Rightarrow gof(x) = x$, for all x

$\Rightarrow (gof)'(x) = 1$, for all x

$\Rightarrow g'(f(x))f'(x) = 1$, for all x [Using Chain Rule]

$\Rightarrow g'(f(x)) = \dfrac{1}{f'(x)}$, for all x

$\Rightarrow g'(f(c)) = \dfrac{1}{f'(c)}$ [Putting $x = c$]

139 (d)

Given, $f(x) = \begin{cases} x^p \cos\left(\dfrac{1}{x}\right), & x \neq 0 \\ 0, & x = 0 \end{cases}$

Since, $f(x)$ is differentiable at $x = 0$, therefore it is continuous at $x = 0$

$\therefore \lim\limits_{x \to 0} f(x) = f(0) = 0$

$\Rightarrow \lim\limits_{x \to 0} x^p \cos\left(\dfrac{1}{x}\right) = 0 \Rightarrow p > 0$
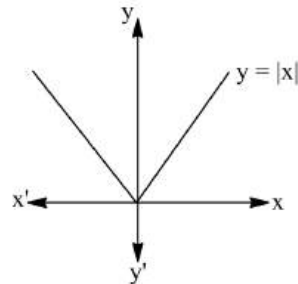
As $f(x)$ is differentiable at $x = 0$

$\therefore \lim\limits_{x \to 0} \dfrac{f(x) - f(0)}{x - 0}$ exists finitely

$\Rightarrow \lim\limits_{x \to 0} \dfrac{x^p \cos\frac{1}{x} - 0}{x}$ exists finitely

$\Rightarrow \lim\limits_{x \to 0} x^{p-1} \cos\dfrac{1}{x} - 0$ exists finitely

$\Rightarrow p - 1 > 0 \Rightarrow p > 1$

140 (a)

It is clear from the graph that $f(x)$ is continuous everywhere and also differentiable everywhere except at $x = 0$

141 (c)

We know that the function

$\phi(x) = (x - a)^2 \sin\left(\dfrac{1}{x - a}\right)$

Is continuous and differentiable at $x = a$ whereas the function $\Psi(x) = |x - a|$ is everywhere continuous but not differentiable at $x = a$

Therefore, $f(x)$ is not differentiable at $x = 1$

142 (d)

$\lim\limits_{x \to 0} \dfrac{2^x - 2^{-x}}{x} = \lim\limits_{x \to 0} 2^x \log 2 + 2^{-x} \log 2$
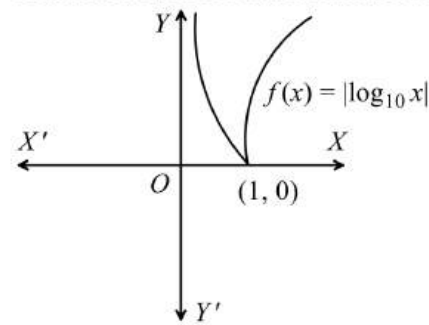
[by L' Hospital's rule]

$= \log 4$

Since, the function is continuous at $x = 0$

$\therefore f(0) = \lim\limits_{x \to 0} f(x) \Rightarrow f(0) = \log 4$

143 (a)

As is evident from the graph of $f(x)$ that it is continuous but not differentiable at $x = 1$

Now,

$f''(1^+) = \lim\limits_{x \to 1^+} \dfrac{f(x) - f(1)}{x - 1}$

$\Rightarrow f''(1^+) = \lim\limits_{h \to 0} \dfrac{f(1 + h) - f(1)}{h}$

$\Rightarrow f''(1^+) = \lim\limits_{h \to 0} \dfrac{\log_{10}(1 + h) - 0}{h}$

$\Rightarrow f''(1^+) = \lim\limits_{h \to 0} \dfrac{\log(1 + h)}{h \cdot \log_e 10} = \dfrac{1}{\log_e 10} = \log_{10} e$

$f''(1^-) = \lim\limits_{x \to 1^+} \dfrac{f(x) - f(1)}{x - 1}$

$\Rightarrow f''(1^-) = \lim\limits_{h \to 0} \dfrac{f(1 - h) - f(1)}{h}$

$\Rightarrow f''(1^-) = \lim\limits_{h \to 0} \dfrac{\log_{10}(1 - h)}{h} = \lim\limits_{h \to 0} \dfrac{\log_e(1 - h)}{h \log_e 10}$

$= -\log_{10} e$

144 (a)

We have,

$f'(x) = \lim\limits_{h \to 0} \dfrac{f(x + h) - f(x)}{h}$

$\Rightarrow f'(x) = \lim\limits_{h \to 0} \dfrac{f(x) + f(h) - f(x)}{h}$ $[\because f(x + y) = f(x) + f(y)]$

$\Rightarrow f'(x) = \lim\limits_{h \to 0} \dfrac{f(h)}{h}$

$\Rightarrow f'(x) = \lim\limits_{h \to 0} \dfrac{\sin h \; g(h)}{h} = \lim\limits_{h \to 0} \dfrac{\sin h}{h} \lim\limits_{h \to 0} g(h)$

$= g(0) = k$

145 (a)

We have,

$f(x) = |x| + |x - 1| = \begin{cases} -2x + 1, & x < 0 \\ 1, & 0 \leq x < 1 \\ 2x - 1, & 1 \leq x \end{cases}$

Clearly, $\lim\limits_{x \to 1^-} f(x) = \lim\limits_{x \to 1} 1 = 1$, $\lim\limits_{x \to 1^+} f(x) = \lim\limits_{x \to 1}(2x - 1) = 1$

and, $f(1) = 2 \times 1 - 1 = 1$

$\therefore \lim\limits_{x \to 1^-} f(x) = \lim\limits_{x \to 1^+} f(x) = f(1)$

So, $f(x)$ is continuous at $x = 1$

Now, $\lim\limits_{x\to 1^-}\dfrac{f(x)-f(1)}{x-1}=\lim\limits_{h\to 0}\dfrac{f(1-h)-f(1)}{-h}=\lim\limits_{h\to 0}\dfrac{1-1}{-h}=0$

and,

$\lim\limits_{x\to 1^+}\dfrac{f(x)-f(1)}{x-1}=\lim\limits_{h\to 0}\dfrac{f(1+h)-f(1)}{h}$

$\Rightarrow \lim\limits_{x\to 1^+}\dfrac{f(x)-f(1)}{x-1}=\lim\limits_{h\to 0}\dfrac{2(1+h)-1-1}{h}=2$

$\therefore$ (LHD at $x=1$) $\ne$ (RHD at $x=1$)

So, $f(x)$ is not differentiable at $x=1$

146 (d)

The given function is differentiable at all points except possibly at $x=0$

Now,

(RHD at $x=0$)

$=\lim\limits_{h\to 0}\dfrac{f(0+h)-f(0)}{h}$

$=\lim\limits_{h\to 0}\dfrac{\sqrt{h+1}-1}{h^{3/2}}$

$=\lim\limits_{h\to 0}\dfrac{h}{h^{3/2}(\sqrt{h+1}+1)}=\lim\limits_{h\to 0}\dfrac{1}{\sqrt{h}(\sqrt{h+1}+1)}$

$\to\infty$

So, the function is not differentiable at $x=0$

Hence, the required set is $R-\{0\}$

147 (a)

We have,

$f(x)\,f(y)=f(x)+f(y)+f(xy)-2$

$\Rightarrow f(x).f\left(\dfrac{1}{x}\right)=f(x)+f\left(\dfrac{1}{x}\right)+f(1)-2$

$\Rightarrow f(x).f\left(\dfrac{1}{x}\right)$

$=f(x)$

$+f\left(\dfrac{1}{x}\right)\quad\left[\begin{array}{l}\because f(1)=2\text{ (Putting }x=y=1\\\text{ in the given relation)}\end{array}\right]$

$\Rightarrow f(x)=x^n+1$

$\Rightarrow f(2)=2^n+1$

$\Rightarrow 5=2^n+1\quad[\because f(2)=5\text{ (given)}]$

$\Rightarrow n=2$

$\therefore f(x)=x^2+1\Rightarrow f(3)=10$

148 (b)

We have,

$f(x)=\dfrac{1}{2}x-1,\text{ for }0\le x\le\pi$

$\therefore \{f(x)\}=\begin{cases}-1,\text{ for }0\le x<2\\0,\text{ for }2\le x\le\pi\end{cases}$

$\Rightarrow \tan[f(x)]=\begin{cases}\tan(-1)=-\tan(1),0\le x<2\\\tan 0=0,\quad\ 2\le x\le\pi\end{cases}$

It is evident from the definition of $\tan[f(x)]$ that

$\lim\limits_{x\to 2^-}\tan[f(x)]=-\tan 1$ and, $\lim\limits_{x\to 2^+}\tan[f(x)]=0$

So, $\tan[f(x)]$ is not continuous at $x=2$

Now,

$f(x)=\dfrac{1}{2}x-1\Rightarrow f(x)=\dfrac{x-2}{2}\Rightarrow\dfrac{1}{f(x)}=\dfrac{2}{x-2}$

Clearly, $f(x)$ is not continuous at $x=2$

So, $\tan[f(x)]$ and $\tan\left[\dfrac{1}{f(x)}\right]$ both are discontinuous at $x=2$

149 (c)

$\lim\limits_{x\to 0}(1+x)^{\cot x}=\lim\limits_{x\to 0}\left\{(1+x)^{\frac{1}{x}}\right\}^{x\cot x}$

$=\lim\limits_{x\to 0}\ e^{x\cot x}=e$

Since $f(x)$ is continuous at $x=0$

$\therefore\quad f(0)=\lim\limits_{x\to 0}f(x)=e$

150 (b)

LHL$=\lim\limits_{h\to 0}f\left(\dfrac{\pi}{4}-h\right)$

$=\lim\limits_{h\to 0}\dfrac{\tan\left(\dfrac{\pi}{4}-h\right)-\cot\left(\dfrac{\pi}{4}-h\right)}{\dfrac{\pi}{4}-h-\dfrac{\pi}{4}}$

$=\lim\limits_{h\to 0}\dfrac{-\sec^2\left(\dfrac{\pi}{4}-h\right)-\text{cosec}^2\left(\dfrac{\pi}{4}-h\right)}{-1}=4$

[by L 'Hospital's rule]

Since, $f(x)$ is continuous at $x=\dfrac{\pi}{4}$, then LHL$=$

$f\left(\dfrac{\pi}{4}\right)$

$\therefore\quad a=4$

151 (a)

If $-1\le x<0$, then

$f(x)=\int\limits_{-1}^{x}|t|\,dt=\int\limits_{-1}^{x}-t\,dt=-\dfrac{1}{2}(x^2-1)$

If $x\ge 0$, then

$f(x)=\int\limits_{-1}^{0}-t\,dt+\int\limits_{-1}^{x}-t\,dt=\dfrac{1}{2}(x^2+1)$

$\therefore f(x)=\begin{cases}-\dfrac{1}{2}(x^2-2),&-1\le x<0\\[2mm]\dfrac{1}{2}(x^2+1),&0\le x\end{cases}$

It can be easily seen that $f(x)$ is continuous at $x=0$

So, it is continuous for all $x>-1$

Also, $Rf'(0)=0=Lf'(0)$

So, $f(x)$ is differentiable at $x=0$

$\therefore f'(x)=\begin{cases}-x,&-1<x=0\\0,&x=0\\x,&x>0\end{cases}$

Clearly, $f'(x)$ is continuous at $x=0$

Consequently, it is continuous for all $x>-1$ i.e. for $x+1>0$

Hence, f and f' are continuous for $x+1>0$

152 (c)

We have,

$$f(x) = \lim_{n \to \infty} \frac{x^{-n} - x^n}{x^{-n} + x^n}$$

$$\Rightarrow f(x) = \lim_{n \to \infty} \frac{1 - x^{2n}}{1 + x^{2n}}$$

$$\Rightarrow f(x) = \begin{cases} \dfrac{1 - 0}{1 + 0} = 1, & \text{if} -1 < x < 1 \\ \dfrac{1 - 1}{1 + 1} = 0, & \text{if } x = \pm 1 \\ \dfrac{0 - 1}{0 + 1} = -1, & \text{if } |x| > 1 \end{cases}$$

Clearly, $f(x)$ is discontinuous at $x = \pm 1$

153 (b)

Clearly, $\log |x|$ is discontinuous at $x = 0$

$f(x) = \dfrac{1}{\log |x|}$ is not defined at $x = \pm 1$

Hence, $f(x)$ is discontinuous at $x = 0, 1, -1$

154 (a)

For continuity, $\lim\limits_{x \to 0} f(x) = k$

$$\Rightarrow \lim_{x \to 0} \frac{\sin 3x}{\sin x} = k \Rightarrow \lim_{x \to 0} \frac{\sin 3x}{3x} \cdot \frac{3x}{\sin 3x} = k$$

$$\Rightarrow 3 = k$$

155 (b)

Since, the function $f(x)$ is continuous

$\therefore \quad f(0) = \text{RHL } f(x) = \text{LHL} f(x)$

Now, RHL $f(X) = \lim\limits_{h \to 0} \dfrac{\log(1+0+h)+\log(1-0-h)}{0+h}$

$$= \lim_{h \to 0} \frac{\log(1 + h) + \log(1 - h)}{h}$$

$$= \lim_{h \to 0} \frac{\frac{1}{1+h} - \frac{1}{1-h}}{1} = 0$$

[by L 'Hospital's rule]

$\therefore \quad f(0) = \text{RHL } f(x) = 0$

156 (d)

$$f(x) = \begin{cases} \dfrac{x-4}{|x-4|} + a, & x < 4 \\ a + b, & x = 4 \\ \dfrac{x-4}{|x-4|} + b, & x > 4 \end{cases} = \begin{cases} -1 + a, x < 4 \\ a + b \\ 1 + b, x > 4 \end{cases}$$

LHL $= \lim\limits_{x \to 4^-} f(x) = a - 1$

RHL $= \lim\limits_{x \to 4^+} f(x) = 1 + b$

Since, LHL=RHL=$f(4)$

$\Rightarrow a - 1 = a + b = b + 1$

$a = 1$ and $b = -1$

157 (d)

We have,

$$f(x) = \begin{cases} \dfrac{-1}{x-1}, & 0 < x < 1 \\ \dfrac{1-1}{x-1} = 0, & 1 < x < 2 \\ 0, & x = 1 \end{cases}$$

Clearly, $\lim\limits_{x \to 1^-} f(x) \to -\infty$ and $\lim\limits_{x \to 1^+} f(x) = 0$

So, $f(x)$ is not continuous at $x = 1$ and hence it is not differentiable at $x = 1$

158 (d)

$$\lim_{x \to \frac{\pi}{4}} f(x) = \lim_{x \to \frac{\pi}{4}} \frac{1 - \sqrt{2} \sin x}{\pi - 4x}$$

$$= \lim_{x \to \frac{\pi}{4}} \frac{-\sqrt{2} \cos x}{4} = \frac{1}{4} \quad \text{[by L 'Hospital's rule]}$$

Since, $f(x)$ is continuous at $x = \dfrac{\pi}{4}$

$\therefore \lim\limits_{x \to \frac{\pi}{4}} f(x) = f\left(\dfrac{\pi}{4}\right) \Rightarrow \dfrac{1}{4} = a$

159 (d)

LHL $= \lim\limits_{x \to 1^-} f(x) = \lim\limits_{h \to 0} 1 - h + a = 1 + a$

RHL $= \lim\limits_{x \to 1^+} f(x) = \lim\limits_{h \to 0} 3 - (1 + h)^2 = 2$

For $f(x)$ to be continuous, LHL=RHL

$\Rightarrow 1 + a = 2 \quad \Rightarrow \quad a = 1$

160 (b)

LHL $= \lim\limits_{h \to 0} \dfrac{\cos 3(0-h) - \cos(0-h)}{(0-h)^2}$

$$= \lim_{h \to 0} \frac{\cos 3h - \cos h}{h^2}$$

$$= \lim_{h \to 0} \frac{-3 \sin 3h + \sin h}{2h}$$

$$= \lim_{h \to 0} \frac{-9 \cos 3h + \cos h}{2} = \frac{-9 + 1}{2} = -4$$

$\therefore \lim\limits_{x \to 0^-} f(x) = f(0) \Rightarrow \lambda = -4$

161 (c)

LHL $= \lim\limits_{x \to a^-} \dfrac{x^3 - a^3}{x - a} = \lim\limits_{h \to 0} \dfrac{(a-h)^3 - a^3}{a - h - a}$

$$= \lim_{h \to 0} \frac{(a-h-a)\{(a-h)^2 + a^3 + a(a-h)\}}{-h}$$

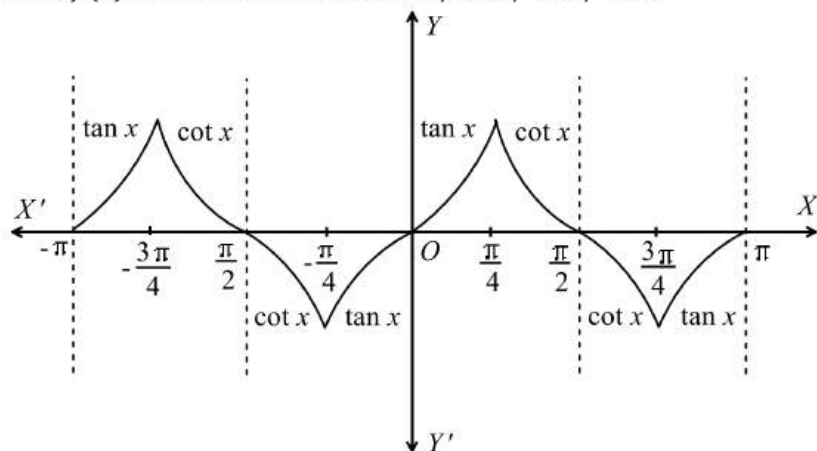$$= 3a^2$$

Since, $f(x)$ is continuous at $x = a$

$\therefore \quad \text{LHL} = f(a)$

$\Rightarrow \quad 3a^2 = b$

163 (a)

We have,

$$f(x) = \begin{cases} \tan x, & 0 \le x \le \pi/4 \\ \cot x, & -\pi/4 \le x \le \pi/2 \\ \tan x, & \pi/2 < x \le 3\pi/4 \\ \cot x, & 3\pi/4 \le x < \pi \end{cases}$$

Since $\tan x$ and $\cot x$ are periodic functions with period π. So, $f(x)$ is also periodic with period π

It is evident from the graph that $f(x)$ is not continuous at $x = \pi/2$. Since $f(x)$ is periodic with period π. So, it is not continuous at $x = 0, \pm\pi/2, \pm\pi, \neq 3\pi/2$

Also, $f(x)$ is not differentiable $x = \pi/4, 3\pi/4, 5\pi/4$ etc

164 (c)

We have,

$f(x) = \{|x| - |x - 1|\}^2$

$\Rightarrow f(x) = \begin{cases} (-x + x - 1)^2, & \text{if } x < 0 \\ (x + x - 1)^2, & \text{if } 0 \le x < 1 \\ (x - x + 1)^2, & \text{if } x \ge 1 \end{cases}$

$\Rightarrow f(x) = \begin{cases} 1, & \text{if } x < 0 \\ (2x - 1)^2, & \text{if } 0 < x < 1 \\ 1, & \text{if } x \ge 1 \end{cases}$

$\Rightarrow f'(x) = \begin{cases} 0, & \text{if } x < 0 \text{ or if } x > 1 \\ 4(2x - 1), & \text{if } 0 < x < 1 \end{cases}$

165 (b)

We have,

$f'(x_0) = \lim\limits_{x \to x_0} \dfrac{f(x) - f(x_0)}{x - x_0}$

$\Rightarrow f'(x_0) = \lim\limits_{x \to x_0} \dfrac{(x - x_0)\phi(x) - 0}{(x - x_0)}$

$\Rightarrow f'(x_0) = \lim\limits_{x \to x_0} \phi(x) = \phi(x_0) \quad [$

$\qquad \because \phi(x) \text{ is continuous at } x = x_0]$

166 (b)

Since, $\lim\limits_{x \to 2^+} f(x) = f(2) = k$

$\Rightarrow \quad k = \lim\limits_{h \to 0} f(2 + h)$

$\Rightarrow \quad k = \lim\limits_{h \to 0} \left[(2 + h)^2 + e^{\frac{1}{2-(2+h)}}\right]^{-1}$

$\Rightarrow \quad \lim\limits_{h \to 0} \left[4 + h^2 + 4h + e^{-1/h}\right]^{-1} = \dfrac{1}{4}$

167 (c)

For $f(x)$ to be continuous at $x = \pi/2$, we must have

$\lim\limits_{x \to \pi/2} f(x) = f(\pi/2)$

$\Rightarrow \lim\limits_{x \to \pi/2} \dfrac{1 - \sin x}{(\pi - 2x)^2} \cdot \dfrac{\log \sin x}{\log(1 + \pi^2 - 4\pi x + 4x^2)} = k$

$\Rightarrow \lim\limits_{h \to 0} \dfrac{1 - \cos h}{4h^2} \times \dfrac{\log \cos h}{\log(1 + 4h^2)} = k$

$\Rightarrow \lim\limits_{h \to 0} \dfrac{1 - \cos h}{4h^2} \times \dfrac{\log\{1 + \cos h - 1\}}{\cos h - 1}$

$\qquad \times \dfrac{4h^2}{\log(1 + 4h^2)} \times \dfrac{\cos h - 1}{4h^2} = k$

$\Rightarrow -\lim\limits_{h \to 0} \left(\dfrac{1 - \cos h}{4h^2}\right)^2 \dfrac{\log(1 + (\cos h - 1))}{\cos h - 1}$

$\qquad \times \dfrac{4h^2}{\log(1 + 4h^2)} = k$

$\Rightarrow -\lim\limits_{h \to 0} \left(\dfrac{\sin^2 h/2}{2h^2}\right)^2 \dfrac{\log(1 + (\cos h - 1))}{\cos h - 1}$

$\qquad \times \dfrac{4h^2}{\log(1 + 4h^2)} = k$

$\Rightarrow -\dfrac{1}{64} \lim\limits_{h \to 0} \left(\dfrac{\sin h/2}{h/2}\right)^4 \dfrac{\log(1 + (\cos h - 1))}{\cos h - 1}$

$\qquad \times \dfrac{4h^2}{\log(1 + 4h^2)} = k$

$\Rightarrow -\dfrac{1}{64} = k$

168 (c)

LHL$= \lim\limits_{h \to 0} f(0 - h) = \lim\limits_{h \to 0} \dfrac{\sin 5(0 - h)}{(0 - h)^2 + 2(0 - h)}$

$= -\lim\limits_{h \to 0} \dfrac{\dfrac{\sin 5h}{5h}}{\dfrac{1}{5}(h - 2)} = \dfrac{5}{2}$

Since, it is continuous at $x = 0$, therefore LHL$= f(0)$

$\Rightarrow \dfrac{5}{2} = k + \dfrac{1}{2} \quad \Rightarrow \quad k = 2$

169 (a)

Since $f(x)$ is continuous at $x = 0$

$\therefore \lim\limits_{x \to 0} f(x) = f(0) = 0$

$\Rightarrow \lim_{x\to 0} x^n \sin\left(\frac{1}{x}\right) = 0 \Rightarrow n > 0$

$f(x)$ is differentiable at $x = 0$, if

$\lim_{x\to 0} \frac{f(x) - f(0)}{x - 0}$ exists finitely

$\Rightarrow \lim_{x\to 0} \frac{x^n \sin\frac{1}{x} - 0}{x}$ exists finitely

$\Rightarrow \lim_{x\to 0} x^{n-1} \sin\left(\frac{1}{x}\right)$ exists finitely

$\Rightarrow n - 1 > 0 \Rightarrow n > 1$

If $n \le 1$, then $\lim_{x\to 0} x^{n-1} \sin\left(\frac{1}{x}\right)$ does not exist and hence $f(x)$ is not differentiable at $x = 0$

Hence $f(x)$ is continuous but not differentiable at $x = 0$ for $0 < n \le 1$ i.e. $n \in (0, 1]$

170 (b)

Clearly, $f(x)$ is not differentiable at $x = 3$

Now, $\lim_{h\to 3^-} f(x) = \lim_{h\to 0} f(3 - h)$

$= \lim_{h\to 0} |3 - h - 3|$

$= 0$

$\lim_{h\to 3^+} f(x) = \lim_{h\to 0} f(3 + h)$

$= \lim_{h\to 0} |3 + h - 3| = 0$

and $f(3) = |3 - 3| = 0$

$\therefore \quad f(x)$ is continuous at $x = 3$

171 (a)

It can easily be seen from the graphs of $f(x)$ and that both are continuous at $x = 0$

Also, $f(x)$ is not differentiable at $x = 0$ whereas $g(x)$ is differentiable at $x = 0$

172 (c)

We have,

$\lim_{x\to 0^-} f(x) = \lim_{h\to 0} f(0 - h)$

$\qquad\qquad = \lim_{h\to 0} \frac{-\sin(a+1)h - \sin h}{-h}$

$\Rightarrow \lim_{x\to 0^-} f(x) = \lim_{h\to 0} f(0 - h)$

$\qquad\qquad = \lim_{h\to 0} \left\{\frac{\sin(a+1)h}{h} + \frac{\sin h}{h}\right\}$

$\Rightarrow \lim_{x\to 0^-} f(x) = \lim_{h\to 0} f(0 - h) = (a + 1) + 1$

$\qquad\qquad = a + 2$

and, $\lim_{x\to 0^+} f(x) = \lim_{h\to 0} f(0 + h)$

$\Rightarrow \lim_{x\to 0^+} f(x) = \lim_{h\to 0} \frac{\sqrt{h + bh^2} - \sqrt{h}}{b\, h^{3/2}}$

$\Rightarrow \lim_{x\to 0^+} f(x) = \lim_{h\to 0} \frac{h + bh^2 - h}{bh^{3/2}(\sqrt{h + bh^2} - \sqrt{h})}$

$\qquad\qquad = \lim_{h\to 0} \frac{1}{\sqrt{1 + bh} + 1} = \frac{1}{2}$

Since, $f(x)$ is continuous at $x = 0$. Therefore,

$\lim_{x\to 0^-} f(x) = \lim_{x\to 0^+} f(x) = f(0)$

$\Rightarrow a + 2 = \frac{1}{2} = c \Rightarrow c = \frac{1}{2}, a = -\frac{3}{2}$ and b

$\qquad\qquad \in R - \{0\}$

173 (c)

For $f(x)$ to be continuous at $x = 0$, we must have

$\lim_{x\to 0} f(x) = f(0)$

$\Rightarrow \lim_{x\to 0} \frac{(9^x - 1)(4^x - 1)}{\sqrt{2} - \sqrt{2\cos^2 x/2}} = k$

$\Rightarrow \lim_{x\to 0} \frac{(9^x - 1)(4^x - 1)}{\sqrt{2} \cdot 2\sin^2 x/4} = k$

$\Rightarrow \lim_{x\to 0} \frac{16 \times \left(\frac{9^x-1}{x}\right)\left(\frac{4^x-1}{x}\right)}{2\sqrt{2}\left(\frac{\sin x/2}{x/4}\right)^2} = k$

$\Rightarrow \frac{16}{2\sqrt{2}} \log 9 \cdot \log 4 = k = 4\sqrt{2} \log 9 \cdot \log 4$

$\qquad\qquad = 16\sqrt{2} \log 3 \log 2$

174 (b)

Given, $f(x) = [\tan^2 x]$

Now, $\lim_{x\to 0} f(x) = \lim_{x\to 0} [\tan^2 x] = 0$

And $f(0) = [\tan^2 0] = 0$

Hence, $f(x)$ is continuous at $x = 0$

175 (b)

Let, $f(x) = x$

Which is continuous at $x = 0$

Also, $f(x + y) = f(x) + f(y)$

$\Rightarrow \quad f(0 + 0) = f(0) + f(0)$

$= 0 + 0$

$\Rightarrow f(0) = 0$

$f(1 + 0) = f(1) + f(0)$

$\Rightarrow \quad f(1) = 1 + 0$

$\Rightarrow \quad f(1) = 1$

As, it satisfies it.

Hence, $f(x)$ is continous for every values of x

176 (c)

Here, $gof = \begin{cases} e^{\sin x}, & x \ge 0 \\ e^{1-\cos x}, & x \le 0 \end{cases}$

$\therefore$ LHD $= \lim_{h\to 0} \frac{gof(0-h) - gof(h)}{-h}$

$= \lim_{h\to 0} \frac{e^{1-\cos h} - e^{1-\cos h}}{-h} = 0$

RHD $= \lim_{h\to 0} \frac{gof(0+h) - gof(h)}{h}$

$= \lim_{h\to 0} \frac{e^{\sin h} - e^{\sin h}}{h} = 0$

Since, RHD=LHD=0

$\therefore \quad (gof)'(0) = 0$

177 (b)

We have,

$$f(x)\begin{cases}(x+1)^{2-\left(\frac{1}{x}+\frac{1}{x}\right)}=(x+1)^2, & x<0 \\ 0, & x=0 \\ (x+1)^{2-\left(\frac{1}{x}+\frac{1}{x}\right)}=(x+1)^{2-\frac{2}{x}}, & x>0\end{cases}$$

Clearly, $f(x)$ is everywhere continuous except possibly at $x=0$

At $x=0$, we have

$$\lim_{x\to0^-}f(x)=\lim_{x\to0^-}(x+1)^2=1$$

and, $\lim_{x\to0^+}f(x)=\lim_{x\to0}(x+1)^{2-\frac{2}{x}}=\lim_{x\to0}(x+1)^{-2/x}$

$$\Rightarrow \lim_{x\to0^+}f(x)=e^{\lim_{x\to0}-\frac{2}{x}\log(1+x)}=e^{-2}$$

Clearly, $\lim_{x\to0^-}f(x)=\lim_{x\to0^+}f(x)$

So, $f(x)$ is not continuous at $x=0$

178 **(b)**

Since $f(x)$ is continuous at $x=0$. Therefore,

$$\lim_{x\to0}f(x)=f(0)$$

$$\Rightarrow \lim_{x\to0}f(x)=k$$

$$\Rightarrow \lim_{x\to0}\frac{\log(1+ax)-\log(1-bx)}{x}=k$$

$$\Rightarrow a\lim_{x\to0}\frac{\log(1+ax)}{ax}-(-b)\lim_{x\to0}\frac{\log(1-bx)}{-bx}=k$$

$$\Rightarrow a+b=k$$

179 **(c)**

Since $f(x)$ is continuous at $x=0$

$$\therefore f(0)=\lim_{x\to0}f(x)$$

$$\Rightarrow f(0)=\lim_{x\to0}\frac{(27-2x)^{1/3}-3}{9-3(243+5x)^{1/5}}\quad\left[\text{Form}\frac{0}{0}\right]$$

$$\Rightarrow f(0)=\lim_{x\to0}\frac{\frac{1}{3}(27-2x)^{-\frac{2}{3}}(-2)}{-\frac{3}{5}(243+5x)^{-\frac{4}{5}}(5)}$$

$$=\left(-\frac{2}{3}\right)\left(-\frac{1}{3}\right)\frac{3^4}{3^2}=2$$

180 **(d)**

$$\lim_{x\to0}\frac{e^{2x}-1-2x}{x(e^{2x}-1)}$$

$$=\lim_{x\to0}\frac{2e^{2x}-2}{(e^{2x}-1)+2xe^{2x}}\quad[\text{using L 'Hospital rule}]$$

$$=\lim_{x\to0}\frac{4e^{2x}}{4e^{2x}+4xe^{2x}}=1\quad[\text{using L 'Hospital's rule}]$$

Since, $f(x)$ is continuous at $x=0$, then

$$\lim_{x\to0}f(x)=f(0)\quad\Rightarrow\quad 1=f(0)$$

181 **(b)**

If a function $f(x)$ is continuous at $x=a$, then it may or may not be differentiable at $x=a$

$\therefore$ Option (b) is correct

182 **(c)**

Let $f(x)=|x-1|+|x-3|$

$$=\begin{cases}x-1 \quad +x-3 \ ,x\ge3 \\ x-1+3-x, \quad 1\le x<3 \\ 1-x \quad +3-x, \ x\le1\end{cases}$$

$$=\begin{cases}2x-4, x\ge3 \\ 2, 1\le x<3 \\ 4-2x, x\le1\end{cases}$$

At $x=2$, function is

$$f(x)=2$$

$$\Rightarrow f'(x)=0$$

183 **(d)**

We have,

$$f(x)=\begin{cases}(x+1)\,e^{-\left(\frac{1}{x}+\frac{1}{x}\right)}=(x+1), & x<0 \\ (x+1)\,e^{-\left(\frac{1}{x}+\frac{1}{x}\right)}=(x+1)e^{-2/x}, & x>0\end{cases}$$

Clearly, $f(x)$ is continuous for all $x\ne0$

So, we will check its continuity at $x=0$

We have,

$$(\text{LHL at }x=0)=\lim_{x\to0^-}f(x)=\lim_{x\to0}(x+1)=1$$

$$(\text{RHL }at\ x=0)=\lim_{x\to0^+}f(x)=\lim_{x\to0}(x+1)\,e^{-2/x}$$

$$=\lim_{x\to0}\frac{x+1}{e^{2/x}}=0$$

$$\therefore \lim_{x\to0^-}f(x)\ne\lim_{x\to0^+f(x)}$$

So, $f(x)$ is not continuous at $x=0$

Also, $f(x)$ assumes all values from $f(-2)$ to $f(2)$ and $f(2)=3/e$ is the maximum value of $f(x)$

184 **(c)**

Since, it is a polynomial function, so it is continuous for every value of x except at $x=2$

$$\text{LHL}=\lim_{x\to2^-}x-1$$

$$=\lim_{h\to0}2-h-1=1$$

$$\text{RHL}=\lim_{x\to2^+}2x-3=\lim_{h\to0}2(2+h)-3=1$$

And $f(2)=2(2)-3=1$

$\therefore$ LHL+RHL= $f(2)$

Hence, $f(x)$ is continuous for all real values of x

185 **(c)**

Continuity at $x=0$

$$\text{LHL}=\lim_{x\to0^-}\frac{\tan x}{x}=\lim_{h\to0}\frac{-\tan h}{-h}=1$$

$$\text{RHL}=\lim_{x\to0^+}\frac{\tan x}{x}=\lim_{h\to0}\frac{\tan h}{h}=1$$

$\therefore$ LHL=RHL= $f(0)=1$, it is continuous

Differentiability at $x=0$

$$\text{LHD}=\lim_{h\to0}\frac{f(0-h)-f(0)}{-h}=\lim_{h\to0}\frac{\frac{\tan(-h)}{-h}-1}{-h}$$

$$=\lim_{h\to0}\frac{+\frac{h^2}{3}+\frac{2h^4}{15}+\cdots}{-h}=0$$

$$\text{RHD}=\lim_{h\to0}\frac{f(0+h)-f(0)}{h}=\lim_{h\to0}\frac{\frac{\tan h}{h}-1}{h}$$

$$=\lim_{h\to0}\frac{\frac{h^2}{3}+\frac{2h^4}{15}+\cdots}{-h}=0$$

$\therefore$ LHD=RHD

Hence, it is differentiable.

186 (b)

We have,

$\lim_{x \to 1^-} f(x) = \lim_{x \to 1}(x - 1) = 0$

and,

$\lim_{x \to 1^+} f(x) = \lim_{x \to 1}(x^3 - 1) = 0$. Also, $f(1) = 1 - 1 = 0$

So, $f(x)$ is continuous at $x = 1$

Clearly, $(f'(1)) = 3$ and $Rf'(1) = 1$

Therefore, $f(x)$ is not differentiable at $x = 1$

187 (d)

We have,

$$f(x) = \begin{cases} \dfrac{x^2 - x}{x^2 - x} = 1, & \text{if } x < 0 \text{ or } x > 1 \\ -\dfrac{(x^2 - x)}{x^2 - x} = -1, & \text{if } 0 < x < 1 \\ 1, & \text{if } x = 0 \\ -1, & \text{if } x = 1 \end{cases}$$

$\Rightarrow f(x) = \begin{cases} 1, & \text{if } x \le 0 \text{ or } x > 1 \\ -1, & \text{if } 0 < x \le 1 \end{cases}$

Now,

$\lim_{x \to 0^-} f(x) = \lim_{x \to 0} 1 = 1$ and, $\lim_{x \to 0^+} f(x) = \lim_{x \to 0} -1 = -1$

Clearly, $\lim_{x \to 0^-} f(x) \ne \lim_{x \to 0^+} f(x)$

So, $f(x)$ is not continuous at $x = 0$. It can be easily seen that it is not continuous at $x = 1$

188 (b)

We have,

$f(x) = |x - 1| + |x - 3|$

$\Rightarrow f(x) = \begin{cases} -(x - 1) - (x - 3), & x < 1 \\ (x - 1) - (x - 3), & 1 \le x < 3 \\ (x - 1) + (x - 3), & x \ge 3 \end{cases}$

$\Rightarrow f(x) = \begin{cases} -2x + 4, & x < 1 \\ 2, & 1 \le x < 3 \\ 2x - 4, & x \ge 3 \end{cases}$

Since, $f(x) = 2$ for $1 \le x < 3$. Therefore $f'(x) = 0$ for all $x \in (1, 3)$

Hence, $f'(x) = 0$ at $x = 2$

189 (d)

We have,

$Lf'(0) = 0$ and $Rf'(0) = 0 + \cos 0° = 1$

$\therefore Lf'(0) \ne Rf'(0)$

Hence, $f'(x)$ does not exist at $x = 0$

190 (c)

Given, $g(x) = \dfrac{(x-1)^n}{\log \cos^m(x-1)}$; $0 < x < 2$, $m \ne 0$, n are integers and $|x - 1| = \begin{cases} x - 1; & x \ge 1 \\ 1 - x; & x < 1 \end{cases}$

The left hand derivative of $|x - 1|$ at $x = 1$ is $p = -1$

Also, $\lim_{x \to 1^+} g(x) = p = -1$

$\Rightarrow \lim_{h \to 0} \dfrac{(1 + h - 1)^n}{\log \cos^m(1 + h - 1)} = -1$

$\Rightarrow \lim_{h \to 0} \dfrac{h^n}{m \log \cos h} = -1$

$\Rightarrow \lim_{h \to 0} \dfrac{n \cdot h^{n-1}}{m \dfrac{1}{\cos h}(-\sin h)} = -1$

[using L 'Hospital's rule]

$\Rightarrow \left(\dfrac{n}{m}\right) \lim_{h \to 0} \dfrac{h^{n-2}}{\left(\dfrac{\tan h}{h}\right)} = 1$

$\Rightarrow n = 2$ and $\dfrac{n}{m} = 1$

$\Rightarrow m = n = 2$

191 (c)

Given, $f(x) = \dfrac{2x^2 + 7}{(x^2 - 1)(x + 3)}$

Since, at $x = 1, -1, -3$, $f(x) = \infty$

Hence, function is discontinuous

193 (a)

LHL $= \lim_{x \to 1^-} f(x) = \lim_{h \to 0}[1 - (1 - h)^2] = 0$

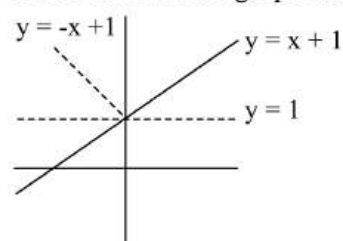RHL $= \lim_{x \to 1^+} f(x) = \lim_{h \to 0}\{1 + (1 + h)^2\} = 2$

Also, $f(1) = 0$

$\Rightarrow$ RHL $\ne$ LHL $= f(1)$

Hence, $f(x)$ is not continuous at $x = 1$

194 (c)

It is clear from the graph that minimum $f(x)$ is

$f(x) = x + 1, \quad \forall x \in R$

Hence, it is a straight line, so it is differentiable everywhere

195 (c)

Since, $f(x)$ is continuous at $x = \dfrac{\pi}{2}$

$\lim_{x \to \frac{\pi}{2}^-}(mx + 1) = \lim_{x \to \frac{\pi}{2}^+}(\sin x + n)$

$\Rightarrow m\dfrac{\pi}{2} + 1 = \sin\dfrac{\pi}{2} + n$

$\Rightarrow \dfrac{m\pi}{2} = n$

196 (a)

This function is continuous at $x = 0$, then

$\lim_{x \to 0} \dfrac{\log_e(1 + x^2 \tan x)}{\sin x^3} = f(0)$

$\Rightarrow \lim_{x \to 0} \dfrac{\log_e\left\{1 + x^2\left(x + \dfrac{x^3}{3} + \ldots\right)\right\}}{x^3 - \dfrac{x^9}{3!} + \dfrac{x^{15}}{5!} - \ldots} = f(0)$

$$\Rightarrow \lim_{x \to 0} \frac{\log_e(1+x^3)}{x^3 - \frac{x^9}{3!} + \frac{x^{15}}{5!} - \cdots} = f(0)$$

[neglecting higher power of x in $x^2 \tan x$]

$$\Rightarrow \lim_{x \to 0} \frac{x^3 - \frac{x^6}{2} + \frac{x^9}{3} - \cdots}{x^3 + \frac{x^9}{3!} + \frac{x^{15}}{5!} - \cdots} = f(0)$$

$$\Rightarrow \quad 1 = f(0)$$

197 (a)

Given, $f(x)$ is continuous at $x = 0$

$\therefore$ Limit must exist

ie, $\lim_{x \to 0} x^p \sin\frac{1}{x} = (0)^p \sin\infty = 0$, when, $0 < p < \infty$ ...(i)

Now, RHD$= \lim_{h \to 0} \frac{h^p \sin\frac{1}{h} - 0}{h} = \lim_{h \to 0} h^{p-1}\sin\frac{1}{h}$

LHD$= \lim_{h \to 0} \frac{(-h)^p \sin\left(-\frac{1}{h}\right) - 0}{-h}$

$= \lim_{h \to 0} (-1)^p h^{p-1}\sin\frac{1}{h}$

Since, $f(x)$ is not differentiable at $x = 0$

$\therefore \quad p \le 1$ ...(ii)

From Eqs.(i) and (iii), $0 < p \le 1$

198 (a)

We have,

$$\lim_{x \to 0} f(x) = \lim_{x \to 0} \frac{\sin x^2}{x} = \lim_{x \to 0} \left(\frac{\sin x^2}{x^2}\right) x = 1 \times 0$$

$$= 0 = f(0)$$

So, $f(x)$ is continuous at $x = 0$. $f(x)$ is also derivable at $x = 0$, because

$$\lim_{x \to 0} \frac{f(x) - f(0)}{x - 0} = \lim_{x \to 0} \frac{\sin x^2}{x} = \lim_{x \to 0} \frac{\sin x^2}{x^2} = 1 \text{ exists finitely}$$

199 (a)

A function f on R into itself is continuous at a point a in R, iff for each $\in > 0$ there exist $\delta > 0$, such that

$$|f(x) - f(a)| < \in \quad \Rightarrow \quad |x - a| < \delta$$

200 (a)

We have,

$$f(x) = x - |x - x^2|, \qquad -1 \le x \le 1$$

$$\Rightarrow f(x) = \begin{cases} x + x - x^2, & -1 \le x < 0 \\ x - (x - x^2), & 0 \le x \le 1 \end{cases}$$

$$\Rightarrow f(x) = \begin{cases} 2x - x^2, & -1 \le x < 0 \\ x^2, & 0 \le x \le 1 \end{cases}$$

Clearly, $f(x)$ is continuous at $x = 0$

Also,

$$\lim_{x \to -1^+} f(x) = \lim_{x \to -1} 2x - x^2 = -2 - 1 = -3$$

$$= f(-1)$$

and,

$$\lim_{x \to 1^-} f(x) = \lim_{x \to 1^-} x^2 = 1 = f(1)$$

So, $f(x)$ is right continuous at $x = -1$ and left continuous at $x = 1$

Hence, $f(x)$ is continuous on $[-1, 1]$

201 (b)

Since $|\sin x|$ and $|e^{|x|}|$ are not differentiable at $x = 0$. Therefore, for $f(x)$ to be differentiable at $x = 0$, we must have $a = 0, b = 0$ and c can be any real number

202 (a)

We have,

$f(u + v) = f(u) + kuv - 2v^2$ for all $u, v \in R$ ...(i)

Putting $u = v = 1$, we get

$f(2) = f(1) + k - 2 \Rightarrow 8 = 2 + k - 2 \Rightarrow k = 8$

Putting $u = x, v = h$ in (i), we get

$$\frac{f(x+h) - f(x)}{h} = kx - 2h$$

$$\Rightarrow \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = kx \Rightarrow f'(x)$$

$$= 8x \quad [\because k = 8]$$

203 (b)

Given, $f(x) = \sin^{-1}\left(\frac{2x}{1+x^2}\right)$

$$\Rightarrow f'(x) = \frac{1}{\sqrt{1 - \left(\frac{2x}{1+x^2}\right)^2}} \times \frac{d}{dx}\left(\frac{2x}{1+x^2}\right)$$

$$= \frac{1 + x^2}{\sqrt{(1+x^2)^2}} \times \frac{2(1 - x^2)}{(1+x^2)^2}$$

$$= \frac{2}{1+x^2} \times \frac{1 - x^2}{|1 - x^2|} = \begin{cases} \dfrac{2}{1+x^2}, & \text{if } |x| < 1 \\ -\dfrac{2}{1+x^2}, & \text{if } |x| > 1 \end{cases}$$

$\therefore f'(x)$ does not exist for $|x| = i$, ie, $x = \pm 1$

Hence, $f(X)$ is differentiable on $R - \{-1, 1\}$

204 (a)

LHL$= \lim_{x \to 0^-} f(x) = \lim_{h \to 0} -h \sin\left(\frac{1}{-h}\right) = 0$

RHL$= \lim_{x \to 0^+} f(x) = \lim_{h \to 0} h \sin\left(\frac{1}{h}\right) = 0$

$\therefore$ LHL=RHL=$f(0)$, it is continuous

LHD$= \lim_{x \to 0^-} f(x) = \lim_{h \to 0}\left[\frac{f(0-h) - f(0)}{-h}\right]$

$= \lim_{h \to 0}\left[\frac{-h \sin\frac{1}{h} - 0}{-h}\right]$ =does not exist

$\Rightarrow f(x)$ is not differentiable at $x = 0$

$\therefore f(x)$ is continuous at $x = 0$ but not differentiable at $x = 0$

205 (b)

Since, $|x - 1|$ is not differentiable at $x = 1$

So, $f(x) = |x - 1|e^x$ is not differentiable at $x = 1$

Hence, the required set is $R - \{1\}$

206 (d)

We have,

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$\Rightarrow f'(x) = \lim_{h \to 0} \frac{f(x)f(h) - f(x)}{h} \qquad [\because f(x+y)$$
$$= f(x)f(y)]$$

$$\Rightarrow f'(x) = f(x) \lim_{h \to 0} \frac{f(h) - 1}{h}$$

$$\Rightarrow f'(x) = f(x) \cdot \lim_{h \to 0} \frac{1 + h\, g\,(h)G(h) - 1}{h}$$

$$\Rightarrow f'(x) = f(x) \cdot \lim_{h \to 0} g(h)\ G(h)$$

$$\Rightarrow f'(x) = f(x) \lim_{h \to 0} G(h) \lim_{h \to 0} g(h) = ab\, f(x)$$